'Zero’ Option in Conjoint Analysis:
Silva Ohannessian, Gilbert Saporta

To cite this version:
Silva Ohannessian, Gilbert Saporta. 'Zero’ Option in Conjoint Analysis: A New Specification of the Indecision and the Refusal - Application to the Video on Demand Market. 2010. hal-02536209

HAL Id: hal-02536209
https://hal-cnam.archives-ouvertes.fr/hal-02536209
Preprint submitted on 8 Apr 2020

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
"ZERO" OPTION IN CONJOINT ANALYSIS:
A new specification of the indecision and the refusal.
Application to the Video on Demand market

Silva OHANNESSIAN¹

Gilbert SAPORTA²

¹Doctorate Student, Conservatoire national des arts et métiers (CNAM), Paris, 5, rue Cavour, 1203 Genève, SUISSE, phone number 0041 22 300 15 90, siloh0@gmail.com
I would like to thank Mr Olivier Monti who has coded and run in Matlab and GiveWin-TSP programs related to the pairs and products reduction, the aggregate estimation model with the maximum likelihood and the bayesian approach, i.e. the calculation of estimates through the mode and the standard deviation, and the graphs.

²Professor in Statistics, Conservatoire national des arts et métiers (CNAM), Paris, 292 rue Saint Martin, 75141 Paris cedex 03, FRANCE, phone number 0033 1 40 27 22 68, fax 0033 1 40 27 25 4, saporta@cnam.fr


**ABSTRACT**

This paper undertakes a study about the "zero" option in conjoint analysis. The "zero" option relates to the no choice of products presented to individuals within the frame of a survey. This no choice embeds two distinct concepts, the refusal and the conflict. The first represents the inappreciation of products, while the second is defined by the preference and choice uncertainty. This work proposes a new econometric specification of the no choice by assuming a mix of utilities maximisation and ordered response models. This mix only associates utilities with products and compares them to the "zero" option thresholds. These comparisons lead to no choice situations without linking utilities to refusal and conflict.

A study on the Video on Demand market has been conducted. The results are obtained by applying a bayesian approach in the case of individual models, and the maximum likelihood in the case of aggregate models. The estimates fit the reality and the significance of the refusal and the conflict demonstrates the importance of these variables in the decision making process.

*Keywords*: "zero" option specification, inappreciation of the products, indecision in the choice, separation and non convergence, bayesian approach, Monte Carlo simulations, market shares
INTRODUCTION

In the conjoint analysis literature, the "zero" option relates to the no choice due to the inappreciation of the scenarios presented within the frame of a survey. However, the econometric specification of this no choice, called refusal, is rarely tackled. Another type of no choice is the indecision in the choice making process resulting from the similarities of the scenarios. This second concept of no choice, labeled conflict, is presented by Tversky and Shafir (1992). The latter demonstrates that the consideration of the conflict disagrees with the utility maximisation theory or the rational theory of the choice.

Taking into account these various elements, we propose a new specification of the "zero" option that integrates the above mentioned concepts, the conflict and the refusal. To respect the unsuitability of the utility maximisation with the no choice, no utilities are associated with neither of the two concepts. Instead, we refer to the notion of ordered response models that governs the no choice. Our modeling of the "zero" option is therefore a mix of specification resulting from the utilities comparison and the ordered response models. This mix associates utilities only with the products and compares them with the "zero" option thresholds. These comparisons allow us to determine the boundaries of no choice situations.

We apply our "zero" option model to the Video on Demand (VoD) market. The obtained results, by using a bayesian approach on the individual models and the likelihood maximisation on the aggregate model, show a good adequation of the model. The estimates are consistent with the reality and the significance of the refusal and the conflict demonstrates their importance in the decision making process. Moreover, the use of a bayesian approach for the parameter estimation gives better results than the widespread penalized likelihood.

SPECIFICATION OF THE "ZERO" OPTION

Definition and Psychological Concepts Associated with the "Zero" Option

The "zero" option as defined in this paper does not totally refer to the notion of no choice described in the literature. While the latter only considers the case of the inappreciation of products in the no choice, we also include in this definition the possibility that individuals have uncertainty in the decision making process. The uncertainty is in fact due to the strong similarities of the goods. Psychological reasons explain this situation of no choice, as we will see below.

Dhar (1997) suggests that one of the frequently mentioned no choice causes is that the consumer tends not to choose goods when the difference of interest between two alternatives is small. This result has also been acknowledged by other authors, like Beattie and Barlas (1992), Festinger (1964) and Janis and Mann (1977) who suggest that a defensive refusal is probably due to the choice difficulties. Tversky and Shafir (1992) pretend that a no choice is more likely in a subset that does not integrate a dominant alternative rather than when there is a clearly superior product. Kuhl (1986) and Sjoberg (1980) notice that it is difficult to maintain the intention to act when there are competitive desires and intentions. Montgomery (1989) states that the individual can abandon or delay his choice if he does not find a dominant structure for a promising alternative. Scholnick and Wing (1988) pretend that a decision situation with a lot of acceptable alternatives in which none is clearly the best, can confuse someone and lead him to inaction. In the recent studies, Baron and Ritov (1994), Ritov and Baron (1990) and Spranca, Minsk and Baron (1991) find a systematic bias towards inaction in the consumer decision making process.
This no choice principle that refers to the difference of interest does not fit the rational theory which assumes that the "zero" option must be chosen when no alternative is attractive or when there is some advantage for additional detailed research (Karni et Schwarz 1977). On the other hand, it matches the psychological research in the field of pre-decision making process that suggests that the consumer refuses to make a decision to avoid a difficult compromise (Tversky and Shafir 1992). Tversky and Shafir (1992) call this situation of no choice conflict. Indeed, according to them the conflict appears when a individual can not make compromise. It results in that some important and insignificant decisions become difficult. The authors also mention the complexity to resolve the conflict due to the uncertainty of the inaction consequences and to the embarrassment caused by the anticipation of the dissonance and the regret.

Tversky and Shafir (1992) are also interested in the differences of theory when the conflict is or is not to be considered. They demonstrate that the consideration of the conflict does not agree with the utility maximisation theory or the rational theory of the choice. This observation is due to the fact that the utility maximisation does not suppose that the "zero" option can be caused by a difficulty in the choice. In fact, the utility maximisation assumes that the consumer only chooses the product with the greatest utility and does not take into account the link between the consumer decision and the conflict caused by the compromise. But the authors declare that the conflict influences the psychological state of the consumer and therefore his choice. In the psychological literature this no choice behavior can be explained by the fact that individuals prefer the inaction consequences than the opposite. Indeed, the uncertain consumer prefers not to choose instead of accepting the choice consequences, e.g. the regret of the purchased product. In addition, one of the inaction consequences, i.e. the no choice, can be the unavailability of the product. The psychological theory pretends that individuals prefer to take the risk not to obtain the product rather than regretting his purchase. As for the utility maximisation, it assumes that this conflict does not influence the no choice because individuals select the option "no alternatives" only when they do not like the products. However, Tversky and Shafir (1992) demonstrate the opposite. Indeed, they show with an application that the rational theory of choice is not respected when there is conflict. In their application, they present alternative pairs to individuals and an additional option that allow them to delay their choice. They prove with this example that the proportion of individuals selecting the additional option increases as the conflict rises. This situation is inverted with the utility maximisation. Therefore, in practice, the utility maximisation principle is not respected in some situations.

The choice approach in the conjoint analysis relies on the utility maximisation to estimate the parameter of the model. But, according to Tversky et Shafir (1992), this principle is inappropriate with the introduction of conflict in the model. And yet some authors, like Elrod, Louviere and Krishnakumar (1992) and Haaijer (1999) who add a series of 0 and/or a constant to the multinomial model, or like Haaijer (1999) who specifies the no choice by means of nested models, base their work on the utility maximisation theory. Therefore, in this paper we model the "zero" option by first including the conflict and then by not associating a utility with the no choice but only with the products.

"Zero" option model, the probabilities and the likelihood function

The model of the no choice presented in the literature uses the principle of utility maximisation to estimate the parameters. Elrod, Louviere and Krishnakumar (1992) specify the no choice as another alternative with the attributes equal to zero and determine the choice between the products and the option "zero" by comparing their utilities. Haaijer (1999) consider approximately the same model than Elrod, Louviere and Krishnakumar (1992) by changing some aspects. He
also suggests an estimation of the no choice by a nested model in two steps with the intention not to suppose the "zero" option as another alternative. Our specification does not use the principle of utility maximisation, it does also not consider the "zero" option as another alternative and is formulated in one step. In fact, it is inspired by the censored regression models (tobit models) that suppose a change of the dependant variable from a certain threshold. In our model, a comparison between the utilities remains, but it only takes place between the products utilities, because the "zero" option is not described by an utility. In fact, our specification mixes the utilities comparison and the ordered response models.

Another aspect of our specification is the no choice caused by the conflict (Tversky and Shafir (1992)), while the literature considers only the case associated with the refusal, i.e. the inappreciation of the products.

For the sake of simplification, we will only consider the case of two products and the "zero" option caused by both the refusal and the conflict. Therefore, four alternatives are presented to the individuals, i.e. the product \( h \), the product \( l \), the refusal, denoted by no products, and the conflict, denoted by both products. Because of the similarities of the products leading to the conflict, we can therefore economically translate this situation by near products utilities. Suppose that an utilities difference of \( \delta' \) does not lead to the conflict and an utility superior to \( \delta \) is the minimum to arouse the interest in the products, then our model that defines the choice of the products \( h \) and \( l \) and the two concepts of the "zero" option, i.e the refusal and the conflict, can be plotted as below:

![Figure 1](image)

**Figure 1**

REPRESENTATION OF THE "ZERO" OPTION MODEL DEFINED BY THE PRODUCTS \( h, l \), THE REFUSAL AND CONFLICT

where \( \delta \) represents the threshold defining the refusal, \( \delta' \) the threshold associated with the conflict and \( u_h \), respectively \( u_l \), the utilities associated with the products \( h \), respectively to the products \( l \). The conjoint analysis literature of the choice model generally expresses the utilities as a linear function. In our "zero" option specification, we also apply this hypothesis to the products utilities. This linearity can be explained by the use of compensatory models in conjoint analysis, that is a negative utility is compensated by a positive one. Therefore, we assume an additive form of the utility function.

According to assumptions set above, the utility \( u_h \), respectively \( u_l \), is formulated as \( u_h = \beta' x_h + \varepsilon_h \), respectively \( u_l = \beta' x_l + \varepsilon_l \) and the errors \( \varepsilon_h \), respectively \( \varepsilon_l \), are normal with a
zero mean and a $\sigma^2$ variance.

This illustration and these assumptions allow us to express the structure of the probabilities associated with each area and used in the log-likelihood function to estimate the model parameters. According to the "zero" option representation, these probabilities can be expressed as follows:

$$P(\text{Choice product } h) = P(u_h \geq \delta, u_h \geq u_l + \delta')$$  \hfill (1)

$$P(\text{Choice product } l) = P(u_l \geq \delta, u_h \leq u_l - \delta')$$  \hfill (2)

$$P(\text{Refus}) = P(y = 3) = P(u_h < \delta, u_l < \delta)$$  \hfill (3)

$$P(\text{Conflict}) = 1 - P(\text{Choice product } h) - P(\text{Choice product } l) - P(\text{Refus})$$  \hfill (4)

These equations show us that we must know the joint distribution of the errors $\varepsilon_h$ and $\varepsilon_l$ to proceed. We suppose a bivariate normal distribution and a covariance equal to zero.

The final expressions of the probabilities associated with our "zero" option model become:

$$P_h = P(\text{Choice product } h) = 1 - \Phi \left( \frac{\delta - \beta'x_h}{\sigma} \right) - \Phi \left( \frac{\beta'(x_l - x_h) + \delta'}{\sqrt{2}\sigma} \right)$$
$$+ \Phi \left( \frac{\beta'(x_l - x_h) + \delta'}{\sqrt{2}\sigma}, \frac{\delta - \beta'x_h}{\sigma}; \frac{1}{\sqrt{2}} \right)$$

$$P_l = P(\text{Choice product } l) = \Phi \left( \frac{\beta'(x_l - x_h) - \delta'}{\sqrt{2}\sigma} \right)$$
$$- \Phi \left( \frac{\delta - \beta'x_l}{\sigma}, \frac{\beta'(x_l - x_h) - \delta'}{\sqrt{2}\sigma}; -\frac{1}{\sqrt{2}} \right)$$

$$P_R = P(\text{Refusal}) = \Phi \left( \frac{\delta - \beta'x_h}{\sigma} \right) \Phi \left( \frac{\delta - \beta'x_l}{\sigma} \right)$$

$$P_C = P(\text{Conflict}) = \Phi \left( \frac{\delta - \beta'x_h}{\sigma} \right) + \Phi \left( \frac{\beta'(x_l - x_h) + \delta'}{\sqrt{2}\sigma} \right)$$
$$- \Phi \left( \frac{\beta'(x_l - x_h) + \delta'}{\sqrt{2}\sigma}, \frac{\delta - \beta'x_h}{\sigma}; \frac{1}{\sqrt{2}} \right)$$
$$- \Phi \left( \frac{\delta - \beta'x_h}{\sigma} \right) \Phi \left( \frac{\delta - \beta'x_l}{\sigma} \right)$$
$$+ \Phi \left( \frac{\delta - \beta'x_l}{\sigma}, \frac{\beta'(x_l - x_h) - \delta'}{\sqrt{2}\sigma}; -\frac{1}{\sqrt{2}} \right)$$

With these probabilities, we can express the likelihood function that allows us to estimate the model parameters:

$$L = \prod_{s=1}^{S} P_{h_s}^{y_{hs}} P_{l_s}^{y_{ls}} P_{R_s}^{y_{Rs}} P_{C_s}^{y_{Cs}}$$  \hfill (5)
where

\[ y_{hs} = \begin{cases} 
1 & \text{when the individual chooses the product } h \text{ in the subset } s \\
0 & \text{otherwise}
\end{cases} \]

\[ y_{ls} = \begin{cases} 
1 & \text{when the individual chooses the product } l \text{ in the subset } s \\
0 & \text{otherwise}
\end{cases} \]

\[ y_{Rs} = \begin{cases} 
1 & \text{when the consumer does not like any product in the subset } s \\
0 & \text{otherwise}
\end{cases} \]

\[ y_{Cs} = \begin{cases} 
1 & \text{when the individual is in a conflict situation when facing the products in the subset } s \\
0 & \text{otherwise}
\end{cases} \]

The estimates result from the maximisation of the log-likelihood function, i.e. when its derivative according to the model parameters equals 0. Because of an identification problem, we have to normalize one of the coefficients. Since the estimates remain true up to a scale parameter, we can set \( \sigma \) to 1. With this normalization, we define the estimates given a scale parameter.

**Separation and non convergence of the maximum likelihood**

Non linear models with the qualitative dependant variable, like the probit and the logit models, use the method of the maximum likelihood (ML) to estimate the unknown parameters. This method does not always end with a solution according to the data. The existence of a solution is therefore not assured. The absence of solution is frequently observed in small data size, i.e. inferior to 50 observations.

In the conjoint analysis approach, it is frequent to make estimations in the case of small sample. This small data size and the use of a ML estimation in the choice models often result in the absence of solution. This kind of problem does not occur with rating and ranking data because of the use of ordinary least square (OLS). The choice data estimation by a probit or a logit model can be either aggregate, or individual. In the aggregate case, the data are large enough and reveal seldom problems about the existence of solutions. However, the individual estimation is carried out with a small data size, since the observations represent the number of products choices per individual. Therefore, because we have to satisfy a proportion of reliable responses, it is essential to restrict the number of observations. This kind of sample in the case of individual analysis of preference by a probit or a logit model frequently leads to divergent estimation of one or several parameters. This divergence of parameters is called the separation.

In addition to the small data size issue, other reasons that lead to the separation exist : the presence of some independent variables with a high predictive value toward the dependant variable and the small ratio between the number of observations and the parameters(inferior to 10). An example of the last reason is the following ratio, \( \frac{\text{number of observations}}{\text{number of parameters}} = \frac{50}{10} = 5 \) that is inferior to 10. In fact, this last reason tends to increase the estimation bias of the ML parameter according to Bull, Mak and Greenwood (2002). As Firth (1993) mentions in his paper, the ML estimation bias is generally of the order of \( O(n^{-1}) \). As we will see later, Firth (1993) proposes a method, called the penalized likelihood, to remove this bias, later used by Heinze and Schemper (2002) to resolve the non convergence problems of the ML method associated with the separation. The non convergence of the maximum likelihood estimation is not always noticed in most of the softwares. In fact, because of the separation, the estimates could be infinite or very large. It is explained by the monotonicity of the log-likelihood function. Some softwares only take into account the convergence of the log-likelihood function despite the infinite estimates
and therefore declare the convergence of the model, while it is not the case.

Solutions: bayesian approach and penalized likelihood

As mentioned before, there are some solutions to solve the separation issues. In this subsection, we will only describe the two most widely used methods, i.e. the bayesian approach and the penalized likelihood.

The penalized likelihood estimation is almost always described for case of the dichotomous logit model, later extended to multinomial logit models (Bull, Mak and Greenwood, 2002). However, because our specification relates to a multinomial probit model, nothing guarantees us that the penalized likelihood is the best alternative to the separation problems. Therefore we also take into consideration the bayesian approach.

The bayesian approach functions on the principle of the conditional distributions stated by Bayes. If \( Y \) is the dependent variable of the model, \( X \) the independent variable matrix and \( \beta \) the unknown parameters, the Bayes principle defines the conditional law of \( \beta \) as :

\[
\text{law of } (\beta|Y) = \frac{\text{law of } (Y|\beta) \cdot \text{law of } (\beta)}{\text{law of } Y}
\]

or mathematically,

\[
f(\beta|Y) = \frac{f(Y|\beta)f(\beta)}{f(Y)} \quad (6)
\]

The use of the Bayes approach in a regression model allows to determine the conditional distribution of the parameter \( \beta \) from which we can deduce an estimate. Therefore, since the density of \( Y \) does not depend on the unknown parameters, \( f(\beta|Y) \) becomes proportional to the following expression :

\[
f(\beta|Y) \propto f(Y|\beta)f(\beta) \quad (7)
\]

In our specification of the ”zero” option we can reformulate the distribution of \( Y \) conditional on \( \beta \) as a function of probabilities, since our dependant variable is qualitative. This conditional function in the general multinomial model with a dependent variable with \( H \) levels can be written in the following way :

\[
P(Y|\beta) = \prod_{i=1}^{n} \prod_{h=1}^{H} P_{hi}
\]

where \( P_{hi} = P(y_i = h) \) et \( y_i = \begin{cases} 1 & \text{if the individual } i \text{ choose the alternative } h \\ 0 & \text{otherwise} \end{cases} \)

Then, the law of \( \beta \) conditional on \( Y \) become :

\[
f(\beta|Y) \propto P(Y|\beta)f(\beta) = \prod_{i=1}^{n} \prod_{h=1}^{H} P_{hi} f(\beta) \quad (9)
\]

To determine the expression of \( f(\beta|Y) \), we have to suppose a distribution for the random coefficients vector \( \beta \), called the prior law. With this in hand, we can express the a posteriori distribution of \( \beta \) conditional on \( Y \), \( f(\beta|Y) \) and then the mean, the mode or the median of this a posteriori distribution that correspond to the various ways to estimate the \( \beta \) parameters vector.
The most frequently prior distributions cited and used in the literature are the normal distribution and the Jeffreys prior.

If the number of unknown parameters of the model with \( n \) observations is great (\( r \) coefficients), then the normal prior distribution of the \( r \) parameters is a multivariate normal law, that is:

\[
f(\beta) = (2\pi)^{-\frac{n}{2}}|\Sigma_{\beta}|^{-\frac{1}{2}} \exp \left\{ -\frac{1}{2} (\beta - \mu_{\beta})'\Sigma_{\beta}^{-1}(\beta - \mu_{\beta}) \right\}
\]  

This formulation supposes to know the mean vector and variance matrix (\( \mu_{\beta} \) and \( \Sigma_{\beta} \)) of \( \beta \). A suggestion given by Congdon (2001) in Galindo-Garre and al. (2004) paper is "that, in absence of prior expectation about the direction or size of covariate effects, flat priors may be approximated in BUGS by taking univariate normal distributions with mean zero and large variance". However, Galindo-Garre and al. (2004) add in their conclusion that this procedure that specifies high variance is not correct with small samples.

Finally, applying the normal prior formulation above to the \( a \ posteriori \) distribution of \( f(\beta|Y) \) gives the following expression:

\[
f(\beta|Y) \propto \prod_{i=1}^{n} \prod_{h=1}^{H} P_{hi}^{y_{hi}} \times (2\pi)^{-\frac{n}{2}}|\Sigma_{\beta}|^{-\frac{1}{2}} \exp \left\{ -\frac{1}{2} (\beta - \mu_{\beta})'\Sigma_{\beta}^{-1}(\beta - \mu_{\beta}) \right\}
\]

Another prior distribution commonly used in a bayesian approach is the Jeffreys prior. In fact, this method has the advantage to be invariant to a transformation of parameters. The principle of the Jeffreys prior is to suppose that the distribution of the \( \beta \) coefficients is proportional to the Fisher information matrix determinant \( |I(\beta)| \):

\[
f(\beta) \propto |I(\beta)|^{\frac{1}{2}}
\]

This matrix is defined as follows:

\[
I(\beta) = -E \left( \frac{\partial^2 \ln P(Y|\beta)}{(\partial \beta)^2} \right)
\]

Since the structure of the function \( \frac{\partial^2 \ln P(Y|\beta)}{(\partial \beta)^2} \) does not depend on \( Y \) in the qualitative variable dependant multinomial model, the mean of this function can therefore be removed:

\[
I(\beta) = - \frac{\partial^2 \ln P(Y|\beta)}{(\partial \beta)^2}
\]

By applying the Jeffreys prior to the multinomial model, we obtain the conditional law \( f(\beta|Y) \) of \( \beta \) that is proportional to:

\[
f(\beta|Y) \propto \prod_{i=1}^{n} \prod_{h=1}^{H} P_{hi}^{y_{hi}} \times \left| \frac{\partial^2 \ln P(Y|\beta)}{(\partial \beta)^2} \right|^{\frac{1}{2}}
\]

The penalized likelihood method, originally developed by Firth (1993) to reduce the bias due to the ML method and later used by Heinze and Schumper (2002) to resolve the estimation problems due to the separation, is in fact the application of the Jeffreys prior in the bayesian approach. Indeed, the Firth (1993) method adds the Fisher information matrix determinant to
the likelihood function in the following way:

\[ L^*(\beta|Y) = L(\beta|Y) |I(\beta)|^{\frac{1}{2}} \]  

(16)

With the penalized log-likelihood function:

\[ \ln (L^*(\beta|Y)) = \ln (L(\beta|Y)) + \frac{1}{2} \ln |I(\beta)| \]  

(17)

that we maximise according to \( \beta \):

\[ \frac{\partial \ln (L^*(\beta|Y))}{\partial \beta} = \frac{\partial \ln (L(\beta|Y))}{\partial \beta} + \frac{1}{2} \left\{ |I(\beta)|^{-1} \left[ \frac{\partial |I(\beta)|}{\partial \beta} \right] \right\} = 0 \]  

(18)

we obtain the estimation of the model.

One of these procedure properties is that it ensures the uniqueness and the existence of a solution because of the strict concavity of the \( \log |I(\beta)| \) and \( L(\beta|Y) \) functions, and because of the upper bound presence on the \( L(\beta|Y) \) function and the lower bound absence on \( \log |I(\beta)| \). This property is valid on condition that the matrix of the explanatory variables \( X \) is of full rank. It is an important property in the case of small samples with separation as it allows to obtain finite estimates.

The expression of the Fisher information matrix and the penalized likelihood method in the case of qualitative dependent variables is essentially formulated in the literature for the dichotomous logit model. Thus, this formulation is not useful for our "zero" option specification. However, Bull, Mak and Greenwood (2002) deal with the multinomial logit case in their paper. Indeed, they extent the Firth (1993) approach to the multinomial logit case by specifying the penalized likelihood model and the Fisher matrix information in the following way:

\[ I(\beta) = (X_M' M X_M) \]  

(19)

where \( X_M = X' \otimes I_H \) and \( M \) is a block diagonal matrix with \( \{m_{ihl}\} \) elements, \( h = 1, \ldots, H \) alternatives, \( l = 1, \ldots, L \) alternatives and \( i = 1, \ldots, n \):

\[ m_{ihl} = \begin{cases} P(y_i = h)(1 - P(y_i = h)) & h = l \\ -P(y_i = h)P(y_i = l) & \text{otherwise} \end{cases} \]  

(20)

The penalized likelihood applied to the multinomial model is defined as:

\[ \frac{\partial \ln (L^*(\beta|Y))}{\partial \beta} = \frac{\partial \ln (L(\beta|Y))}{\partial \beta} - I(\beta)b_1(\hat{\beta}_{MV})^* = 0 \]  

(21)

where \( b_1(\hat{\beta}_{MV})^* \) corresponds to the ML asymptotic bias of order \( n \).

This bias is written in the multinomial model case as:

\[ b_1(\hat{\beta}_{MV})^* = -\frac{1}{2} I(\beta)^{-1} \{X_M' Q (X_M \otimes X_M) vec(I(\beta)^{-1})\} \]  

(22)

where

\[ Q = \sum_i E_i \sum_i (E_i \otimes E_i)' \]

\[ E_i = e_i \otimes I_H, \ e_i \text{ is a vector } n \times 1 \text{ only formed of 0 except for the } i^{\text{th row}} \]

\[ Q_i = \sum_{hlr} q_{hldr}(t_l \otimes t_r)' \]
\[ q_{ihlr} = \begin{cases} 
P(y_i = h)|(1 - P(y_i = h))(1 - 2P(y_i = h)) & h = l = r \\
2P(y_i = h)P(y_i = l)P(y_i = r) & h \neq l \neq r \\
-P(y_i = h)(1 - 2P(y_i = h))P(y_i = r) & h = l \neq r \\
-P(y_i = h)P(y_i = l)(1 - 2P(y_i = r)) & h \neq r \neq h \text{ or } r = l 
\end{cases} \]

and \( \iota_h \) is a vector \( H \times 1 \) with only 0 except for the \( h^{th} \) row.

This methodology of the penalized likelihood maximisation described above is however not implemented in the statistical softwares. Indeed, these programs only propose this approach for the dichotomous logit model. Its application and its implementation in the logit multinomial case is difficult and more complex with the probit multinomial model considering the more complicated expression of the probabilities.

In their paper, Bull, Mak and Greenwood (2002) have implemented their method explained above with the Gauss programming language. But they uses modified iterative equations presented in their paper.

The next section will illustrate our "zero" option specification by applying it to the Video on Demand (VoD) market. It will describe among other things the relevant attributes selected for our VoD model, the implementation of the survey by means of design experiments, the processus of the estimation method used, the estimates and the market shares.

**APPLICATION TO THE VIDEO ON DEMAND MARKET**

*Video on Demand description, relevant characteristics and survey*

The Video on Demand (VoD) is a website or a television platform that allows people to watch paying movies, series, etc. when they want and when they decide. To choose the relevant attributes in the creation of a VoD website, we have done several research concerning some complicated computer notions to better understand the way that VoD websites work. In the end, we have decided to describe the VoD website\(^3\) with the following characteristics:

**Attribute A : The programmes quantity** : 1000 (1), 600 (2)

**Attribute B : The composition of the website according to the movies and the series** : 100\% Movies (1), 75\% Movies 25\% Series (2), 50\% Movies 50\% Series (3), 100\% Series (4)

**Attribute C : The composition of the website according to the novelty of the programmes** : A maximum of movies-series novelties (1)\(^4\), The half of the maximum novelties and the rest in old programmes (2), Only old programmes (3)

**Attribute D : Tariff** : Paying per movies-series (1), Free with advert\(^5\) (2), Subscription (3)

**Attribute E : Video hire length at launch** : 24 hours (1), 48 hours (2)

**Attribute F : Availability of the trailer or the extract** : Trailer available (1), Not available (2)

\(^3\) These attributes and their levels reflect the characteristics of the VoD at the beginning of our study in July to November 2007. In January 2007, the questionnaire with the final products was submitted to the students. We have acknowledged changes in the selected characteristics since then, but not always in the more important ones.

\(^4\) maximum of movies=140 and maximum of series=96 ; these figures represent average numbers among the providers at the time this research was undertaken

\(^5\) except the novelties
With these characteristics we elaborate a experiment design that allows us to select some products or combinations of attributes and present them into pairs. The experiment design used is a D-optimal design. As in Benammou, Saporta and Swissi (2007) a first reduction of the pairs is done. We also remove the unfeasible products and the pairs of no interest. The application of the D-optimal experiment design to the remaining pairs and products provide us the basis to create the 20 choices of the questionnaire that we then submit to our target sample. One of the essential characteristics used to select our sample was that individuals must have good computer skills and a fast Internet connection. Therefore, we concentrate on students because of their access to a powerful connection generally offered by the University, and their assumed computer knowledge. With the target defined, we conduct the survey that provides us the data for the application of our specification. A last filter is then applied to focus only on reliable questionnaire. Finally, we end up with 74 individuals for our application. The estimation method used further and the comments of the estimates are described in the subsequent sections.

Bayesian approach

Because of the data separation and the non convergence of the maximum likelihood estimation of the individual models, we use another alternative than the ML. The two directions considered are the penalized likelihood and the bayesian approach. Since the multinomial probit case of the penalized likelihood is not described in the literature, and that the common statistical softwares do not implement it, we opt for the bayesian approach. However, in the cases where individuals only choose two out of the four alternatives in the questionnaire, we decide to estimate these individual models with the two methods. We later demonstrate that the results associated with the penalized likelihood do not match the VoD reality, while the bayesian approach shows a good adequacy.

As for the aggregate model, we use the maximum likelihood estimation. Since the data size is large enough in the case of the aggregate model, the estimates converge and therefore can give some information on the bayesian approach in the individual case. Indeed, we can use this information to derive the mean vector and the variance matrix of the supposed normal prior distribution. The Jeffreys prior in the bayesian approach is not used in our specification, because, as said above, it is equivalent to the penalized likelihood maximisation that provides poorer results than the bayesian approach in the dichotomous case. Therefore, the estimation of our "zero" option model is made with a bayesian approach. We suppose a normal prior with a mean vector and a variance matrix derived from the aggregate model estimation. The estimation of the model parameters comes from the calculation of the \textit{a posteriori} distribution mode. We know that in fact several criteria can be envisaged to estimate the model parameters, i.e. the mode, the mean and the median. But we opt for the mode, since we search the maximum of the $f(\beta|Y)$ function. The mean is also a good estimator of the maximum if the distribution is symmetric. But, we do not know if it is the case. Finally, we choose the mode because it correspond to the maximum for any distributions.

Since the $f(\beta|Y)$ \textit{a posteriori} distribution is not clearly definable, we use the Monte Carlo simulation to calculate the estimates from the mode. The Monte Carlo simulations applied to our "zero" option specification consist of 10’000 $\beta$ vectors generated from the multivariate normal distribution with the mean vector and the variance matrix given by the aggregate estimation model. These 10’000 samples permit to calculate the values of the $f(\beta|Y)$ \textit{a posteriori} distribution. From all those values, we select the greatest one and we look for the $\beta$ vector that has generated this maximum. The values in this vector correspond to the estimates, i.e. the mode.
To calculate the standard deviation of the parameters without knowing the distribution of the \textit{a posteriori} law, we assume that this standard deviation is equal to the half of the length between the first and the last quantile (25\% and 75\%) of the observations of the \textit{a posteriori} distribution, i.e. the 25\% of the observations above and below the mode. To this end, we generate values from the $f(\beta|Y)$ \textit{a posteriori} distribution that we sort and plot according to each parameter of $\beta$, i.e. the refusal, the conflict thresholds and the parameters associated with each level of attribute. In the appendix 1, we present this graphs for one of the individual models which corresponds to a person who has chosen at least once each of the four alternatives in the questionnaire. Then we locate the values of $f(\beta|Y)$ corresponding to 25\% above and below the $f(\beta|Y)$ values associated with the mode (red lines, appendix 1). We report the values of $\beta$ associated with the $f(\beta|Y)$ of the 25\% below and above the mode (green line, appendix 1) to compute the length of each parameter. The standard deviation corresponds to the half of this length.

The results and the comments of our "zero" option model by a bayesian approach are given in the next subsection.

\textit{Results and comments}

We distinguish the results depending on the overall number of alternatives chosen by an individual throughout the questionnaire. For example, if an individual has only selected the options product $h$, $l$ and the refusal, the dependent variable is then described with three levels rather than four. In this case, the individual has not been in a conflict situation along the 20 questions. Others cases with less than the four levels for the dependent variable also occurred during the survey. We only present the results of one individual belonging to a sub-sample representing each case of the dependent variable levels. The comments for a specific sub-sample are valid for all individuals belonging to it. The individual results are comprised of the estimates, the standard deviations and the importance of each attribute that are calculated according to the following equation:

$$\text{importance of the attribute } i = \frac{(\text{maximum} - \text{minimum})_{\text{of the estimates in the attribute } i}}{\sum_{i}(\text{maximum} - \text{minimum})_{\text{of the estimates in the attribute } i}}$$

In the model where people only choose the product $h$ and $l$ without being in a conflict and a refusal situation, the results of the individual 22 is presented in the appendix 2, table A1.

In this table, we notice that the signs of attribute levels fit expectations. Indeed, the results demonstrate a preference for the hire length of 48 hours, a quantity of 1000 programmes and the free access to videos with adverts. Additionally, a website with a maximum of novelties and a composition of 75\% of movies and 25\% of series prevails in the individual choices. As for the trailer, it is not essential in the selection of a VoD website. The attributes that affect mainly the choice of Video on Demand website are the composition according to movies and series, according to the novelty of the programmes and the tariff. Indeed, according to the percentage of importance, these attributes are determinant in the Video on Demand website selection. On the other hand, the quantity of the programmes, the hire length or the availability of the trailers do not have any importance in the website choice.

The standard deviations are reasonable relative to the estimates, as they are inferior to the estimated values of the unknown parameters. Finally, we can say that as a whole the estimation results with the bayesian approach of this individual are satisfactory.
The results of the other individuals of this type of model are similar. Therefore, all the comments above can be applied to them. It will be identical when we will present and comment the other situations according to the dependent variable levels. The same reasoning applies to the other types of model where the similarities of results across individuals enable us to present and comment tables for only one individual per model type.

Since the model with the explained variable corresponding to the choice of product $h$ and $l$ is dichotomous, we can also apply the penalized likelihood maximisation. As some softwares like S-Plus contain the implementation of this estimation procedure, we decide to test it to compare the results with the bayesian approach since the literature generally recommends it in the case of dichotomous logit model. Thus, we want to know if the properties of the penalized likelihood estimation suit to our "zero" option probit specification.

The penalized likelihood estimates of the individual 22 with S-Plus are in the appendix 2, table A2.

The results derived from the penalized likelihood maximisation are not consistent in terms of signs. For example, the individual 22 shows a negative utility for the level 1000 of the catalogue attribute, while the level 600 is positive. Yet, it is more likely that the individual 22 prefer to have 1000 programmes in a VoD website than 600. Additionally, the individual seems to prefer the paying VoD website to the free one with advert. This observation is also questionable. Also the video hire length does also not reflect the reality, since its sign shows a preference for the level 24h rather than 48h.

Moreover, the standard deviations are higher that the estimates. This fact reveals a bad adequacy of the penalized likelihood approach when applied to our "zero" option specification. Therefore, the bayesian approach is clearly a better choice.

In the model where the four alternatives are at least chosen once, the results of the bayesian method are given for the individual 2 in the appendix 3, table A3.

We generally notice that the signs of the coefficients of this sub-sample as well as their magnitude reflect the VoD reality. Besides, the standard errors are appropriate in relative to the size of the estimates. Indeed, in the case of ordered attributes like the programmes quantity, the hire length and the tariff between a paying and a free website, it seems logical that an individual prefer the greatest number of programmes or hire length as well as a website with free videos rather than having to pay for it. With these bayesian estimates we observe that the level 24h of the attribute hire length, respectively the level 600 of the attribute quantity, is systematically inferior to the level 48h, respectively the level 1000 or negative, while 48h, respectively 1000, is positive. The same assertion is possible for the levels free and paying as well as for the levels free and subscription of the attribute tariff. Indeed, the value of the level free is systematically superior to the level paying, respectively to the level subscription. Additionally, the subscription option does not seem to be preferred to the paying one.

The conflict and the refusal coefficients are significant for all individuals of this sub-sample, since the ratio between the estimates and the standard deviation is large enough (largely greater than
2). Indeed, the ratio of the individual 2 is :

\[
\begin{align*}
\text{refusal ratio} &= \frac{1.271}{0.083} = 15.299 \\
\text{conflict ratio} &= \frac{0.476}{0.036} = 12.929
\end{align*}
\]

The significance of the results for the conflict variable confirms the uncertainty of the behaviour of the consumer facing similar products. Therefore, the introduction of the conflict leads to additional information as to the consumer preferences and thus allows to improve the accuracy of the estimates.

Another general observation concerning this sub-sample is that the attributes with the highest and most significant importance are the composition, the level of novelties and the tariff. Therefore, a website offering movie trailers, a great hire length and a great catalogue will not represent a decisive product for the consumer. On the contrary, a quantity of programmes close to what can be found in a video club, the free movies and series (except for the novelties), or a correct composition of movies and series affect more the consumer behavior in the Video on Demand choice.

These various assertions demonstrate that our ”zero” option model better reflect the real consumer preference and allow us to conclude that the bayesian approach yield consistent and significant results in the case of data separation. Additionally, the conflict alternative in the questionnaire offers people another aspect of the decision making process that exist in reality, since this option is significant and chosen by a number of individuals. The addition of the conflict therefore provides supplementary information to the notion of refusal existing in the literature.

In addition to the two previous model types, the dataset also contains individuals having selected only three options, respectively two, among the four available. The three alternatives model, respectively the two alternatives model, are, either the product \( h \), \( l \) and the refusal, or the product \( h \), \( l \) and the conflict, respectively the product \( h \) and the refusal. We will not comment the results for these sub-samples, as they converge to what has already been said about the four alternative model. The case of the product \( h \) and the refusal option (individual 62) has been supposed consistent because the individual gives the correct answer to the test question (choice 21\(^{6}\)) and that his answers are not random.

Additionally, since the variable of this model is dichotomous, we also try to estimate it with the penalized likelihood maximisation in S-Plus. Unfortunately, this method does not work, since the \( X'X \) matrix is not of full rank.

Finally, we present the results of the aggregate model that we used as an \textit{a priori} information for the bayesian approach. These results come from the application of the maximum likelihood estimation adapted to include each of the models that correspond to the number of alternatives selected in the questionnaire, into a grouped model to get an overall result. In others words, we combine the different models into a unique one to obtain the estimation of the unknown parameters \( \beta = ( d \ d1 \ \beta_1 \ \ldots \ \beta_{10} ) \). Indeed, in this aggregate estimation, we associate the corresponding model and the log-likelihood function with each sub-sample and then make a

\[^{6}\text{This question has an obvious answer and is used to assess the consistency of the other answers.}\]
grouped estimation of all these models. The results of this aggregate "zero" option model estimation is presented in the table A4 of the appendix 4.

The aggregate model estimation is carried out on a homogenous population, since all students are from the same University, in the same degree. Hence, they come from the same social environment and are approximately in the same age bracket. With heterogenous population, we should have first created homogenous groups and estimated the model parameters for each group.

The table A5 of the appendix 5 gives the estimates of the model parameters without the reference level that we have removed to obtain a $(X'X)$ full rank matrix. In this table, we show their t-statistics and their p-values. The estimates of the reference variable are obtained through the assumption associated with the conjoint analysis, i.e. the compensation model that is mathematically translated as the sum of the levels of each attribute equal to zero. The reference variables are in fact the last levels of each attribute:

- catalogue : 600
- website composition : 100% Series
- composition of novelties : only old videos
- tariff : subscription
- hire length : 48 hours
- trailer : none

The aggregate model estimation shows that the concept of "zero" option, the refusal and the conflict, are significant. Indeed, their t-statistics are equal 11.638 for the refusal and 8.825 for the conflict (appendix 5). These values are large enough to conclude with the significance of these parameters, as confirmed by the 0 p-values. Therefore, the conflict and the refusal offer an additional explanation to the consumer preference choices of the VoD market and are not irrelevant in this kind of preference analysis. According to the refusal estimated value, individuals appreciate products with an utility superior to 1.308, and according to the conflict estimate the products with an utility difference inferior to 0.471 lead to an uncertainty in the consumer choice.

The attributes of greatest importance are the website composition, the level of novelties and the tariff. The latter influences significantly the consumer decision making process. The variable of the highest importance is the website composition. Then, comes the level of novelties, and finally the tariff. It is surprising that the individuals does not consider the tariff as the most important characteristic. It means that a well-designed website can be successful even it is paying. However, the magnitude of the level "free with advert" is higher than the others, i.e. the composition and the level of novelties. Therefore, the value associated with the free website has a strong influence in the calculation of the product utility that contains this level.

As for the quantity of programmes in the catalogue, the hire length and the availability of the trailer, they are all not conclusive in the VoD website choice according to their importance value. Additionally, the negative sign of the trailer level shows that this service is more unfavorable than the opposite. It must be noticed however that the importance value of this variable is not significant.

The sign and the magnitude of the estimated coefficients are consistent with the expected reality. Indeed, for ordered attributes like the quantity of programmes and the hire length, the estimates are negative for the lower values and positive for the higher ones. Therefore, the lower value of the hire length (24h), respectively the quantity of programmes (600), is smaller than the highest one, i.e. 48h for the hire length, respectively 1000 for the quantity of programmes. The same
observation is made between the free and the paying or subscription levels. Indeed, it makes sens that the parameters associated with the paying or the subscription tariff are lower than the free video. The estimation results of the coefficients of that attribute tariff reflect this reality.

The estimates for the website composition variable demonstrate preferences in decreasing order for the level "75% Movies and 25% Series", the level "50% Movies and 50% Series", and finally the level "100% Movies". Creating a website with only series does not appeal the VoD consumer.

A maximum of novelties is an important criterion in the website choice. However, half of the novelties available in video clubs is, in terms of satisfaction, close to the maximum of this novelties level. On the other hand, a website with only old programmes does not really reflect the consumer preference in the Video on Demand.

Generally, the estimation results of the "zero" option aggregate model are consistent and satisfactory from a statistical and economic point of view. They reflect well the reality of the Video on Demand preference. They demonstrate the significance and the importance of the uncertainty in the choice of similar products (conflict) as well as the indifference of some other VoD websites (refusal). These results allow finally to create VoD websites that better suit the individual preferences.

In the last subsection, we present the calculation of the market shares for some actual websites and for an ideal one created based on the estimation results.

*Purchase probabilities and market shares*

The aggregate estimation results of our "zero" option model allow us to determine the attribute levels that influence the selection of the VoD website. From this information, we create the ideal website that would satisfy the most. They are made up of the following characteristics:

- Catalogue : 1000 programmes
- Composition : 75% Movies 25% Series
- Novelty : maximum
- Tariff : free advert
- Hire length : 48h
- Availability : none

The aim of the Ideal website creation is in fact to calculate the market shares related to existant VoD websites. The most popular in France with a composition of movies and series only are Canalplay and TF1Vision. We select the levels that describe the Canalplay and TF1Vision websites in order to make them as close as possible to their characteristics from July to November 2006. Some levels are a bit different and inexistant in our level selection, so we opted for the closer one. For example, TF1Vision in summer 2006 proposed one of their own series free. But, since most of the programmes are paying, we decide to select the level paying of the attribute tariff for this website. All the information about the following characteristics are collected from computer magazines and through direct visit of these websites from July 2006 to November 2006. Thus, Canalplay and TF1Vision are composed of the following levels attributes:

**Canalplay**

- Catalogue : 1000 programmes
- Composition : 100% Movies
- Novelty : half
- Tariff : paying
- Hire length : 24h
- Availability : none

**TF1Vision**

- Catalogue : 600 programmes
- Composition : 75% Movies 25% Series
- Novelty : half
- Tariff : paying
- Hire length : 24h
- Availability : trailer

According to these website characteristics we can now calculate the individual purchase probabilities in the first place, and the market shares in the second place. The market shares are deduced either from the individual model or from the aggregate model.

From the individual estimations, we associated with each attribute level for each website their corresponding estimates. The appendix 8 shows an example for a given individual. The same procedure is applied to each individual. Then, we calculate the utility $u_i$ associated with each website $i$ for each individual by summing the corresponding estimates. The utilities values are useful to determine the individual purchase probabilities. To this end, two approaches are used, i.e. the Bradley-Terry-Luce and the logit method. The following equations define them:

Bradley-Terry-Luce :  
$$ u_i \sum_i u_i$$

\[ \text{logit : } \frac{\exp(u_i)}{\sum_i \exp(u_i)} \]  

Thus, we calculate for each individual the purchase probability according to the Bradley-Terry-Luce and logit equations. From these values (74 individuals), we observe that they are generally largely higher for the Ideal website than for the others. Moreover, it seems that both the Canalplay and TF1Vision websites share out evenly the VoD market since their individual purchase probabilities are very similar in a market comprised of these three websites.

With these purchase probabilities per individual, we calculate the market shares of each website by averaging the probabilities:

$$ \text{Market shares} = \left( \frac{\sum \text{purchase probabilities}}{\text{individual number (74)}} \right)$$

The market shares derived from the Bradley-Terry-Luce and logit methods are as follows:

<table>
<thead>
<tr>
<th>Method</th>
<th>Market shares in %</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Canalplay</td>
</tr>
<tr>
<td>Bradley-Terry-Luce</td>
<td>16.479</td>
</tr>
<tr>
<td>logit</td>
<td>7.686</td>
</tr>
</tbody>
</table>

\[ ^7 \text{For example, the Bradley-Terry-Luce probabilities purchases for the individual 2 are : Canalplay } 16.77\%, \text{ TF1Vision } 15.65\% \text{ and Ideal } 67.56\%. \text{ The logit results are : Canalplay } 7.85\%, \text{ TF1Vision } 7.45\% \text{ and Ideal } 84.69\%. \text{ We will not give the values of the others individuals as they are very similar.} \]
We reach the same conclusions as with the individual purchase probabilities, i.e. the Ideal scenario offers much higher market shares than Canalplay or TF1Vision whose respective market shares are very similar.

However, the market shares calculated with the individual purchase probabilities does not take into account the refusal and the conflict. We consider these situations only with the aggregate market share calculation. The reason is that we can only obtain the aggregate utilities of the websites with the aggregate model and compare them to the refusal and conflict thresholds. In the individual case we can not have the correct aggregate utilities considering the incomparable measurement scales.

As for the market shares computed from the aggregate model, we obtain them by calculating the scenario utilities from the aggregate maximum likelihood estimates:

<table>
<thead>
<tr>
<th>Utilities</th>
<th>Canalplay</th>
<th>TF1Vision</th>
<th>Ideal</th>
<th>Refusal</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>δ</td>
<td>δ</td>
<td></td>
<td>δ</td>
</tr>
<tr>
<td></td>
<td>0.749</td>
<td>1.021</td>
<td>3.053</td>
<td>1.308</td>
</tr>
</tbody>
</table>

These estimated utilities can be used to determine the products that are not appreciated by the consumers. On the other hand, the utility differences can be used to detect a conflict situation:

<table>
<thead>
<tr>
<th>Difference</th>
<th>Canalplay-TF1Vision</th>
<th>Canalplay-Ideal</th>
<th>TF1Vision-Ideal</th>
<th>Conflict</th>
</tr>
</thead>
<tbody>
<tr>
<td>δ′</td>
<td>0.271</td>
<td>2.303</td>
<td>2.031</td>
<td>0.471</td>
</tr>
</tbody>
</table>

According to the tabulated values, we notice that the Canalplay and TF1Vision websites are in the refusal area since their utilities are inferior to δ. Therefore, in a market where the Canalplay, TF1Vision and Ideal websites coexist, the Canalplay and TF1Vision products are not selected by the consumers and only the Ideal VoD website represents an interesting option leading to 100% market shares. Additionally, according to the values of the differences, it seems that Canalplay and TF1Vision products create difficulties to decide between them because of their similarities. Indeed, the difference between the Canalplay and TF1Vision websites is inferior to the δ′ value that represents the conflict.

A 100% market share does not provide any information about the Canalplay and TF1Vision influences on the Video on Demand market. So we decide to calculate the purchase influences in a market with only Canalplay and TF1Vision. Because we only compare two scenarios in our market, with the equations (1), (2), (3) and (4) we can directly calculate the Canalplay, TF1Vision, refusal and conflict probabilities.8

By inserting in the above mentioned equations the estimates resulting from the aggregate maximum likelihood estimation, we obtain the following results:

<table>
<thead>
<tr>
<th>Probabilities</th>
<th>Canalplay</th>
<th>TF1Vision</th>
<th>Refusal</th>
<th>Conflict</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.186</td>
<td>0.291</td>
<td>0.436</td>
<td>0.085</td>
</tr>
</tbody>
</table>

We observe that the refusal probability is high but way below 100%, i.e. it does not suppose that the Canalplay and TF1Vision products are not appreciated by the consumers. Indeed, they have market shares superior to 0% when they coexist in a same market. In fact, on 100

---

8where σ is set to 1 to take into account the parameters identification.
individuals, 19 select Canalplay, 29 TF1Vision, 44 do not like the products and 8 do not reach any decision about their website preferences. To take into account refusal and conflict situations, we still must transform these probabilities to obtain the real market shares. Indeed, the 44% in the refusal area does not correspond to potential customers and therefore must be excluded from the market share calculations. The 8% are shared out evenly among the Canalplay and the TF1Vision products because of the consumer uncertainty. Because the refusal probability has been removed, we must readjust the Canalplay and TF1Vision probabilities to obtain a sum of probabilities equal to 100% in the following way:

<table>
<thead>
<tr>
<th>Probability with conflict</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Canalplay</td>
<td>TF1Vision</td>
</tr>
<tr>
<td>0.186 + 0.0428</td>
<td>0.291 + 0.042</td>
</tr>
<tr>
<td>= 0.229</td>
<td>= 0.334</td>
</tr>
</tbody>
</table>

Sum of the probabilities with conflict
0.563

<table>
<thead>
<tr>
<th>Market shares</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canalplay</td>
</tr>
<tr>
<td>0.229</td>
</tr>
<tr>
<td>0.563</td>
</tr>
<tr>
<td>0.407</td>
</tr>
</tbody>
</table>

This method that takes account of the refusal and the conflict shows that in a market where there are only individuals interested in the Canalplay and TF1Vision websites, the market shares reach 40.71% for Canalplay and 59.29% for TF1Vision. The market shares of TF1Vision website are here higher than Canalplay. This result demonstrates that the refusal and the conflict are a source of additional information in the behavioral understanding of the decision making process. Indeed, our "zero" option specification allows us, for the market shares calculation, to take only into account the potential consumers and to insert people who hesitate in their choices. Therefore, our market shares truly represent the consumers that are potentially ready to buy the type of products proposed.

CONCLUSION

In this paper, we present a new specification of the "zero" option defined in the literature as an inappreciation of products (refusal) and that is not tackled in details. Additionally, we add to this refusal a new element, called conflict. The latter takes into account the case of uncertainty during the decision making process due to similarities in the products. Our specification relates to a model of pair comparisons to which we add the two concepts of the "zero" option, i.e. the refusal and the conflict. Indeed, the addition of the refusal, respectively the conflict, is done by adding to the pair comparison the option "I do not like any product", respectively "I like both products". Our model is in fact a mix of the ordered response and the utility comparison models. The utility comparison model is implemented by defining the products with an utility and by comparing them. As for the ordered response model, it is used to describe the refusal and the conflict by thresholds that are compared to the products utilities. The reason of this models mix is that the utility maximisation does not consider the conflict in its estimation process. Additionally, an utility associated with the no choice is not really interpretable as it would be for products.

Because our "zero" option model is specified by a probit choice structure, we estimate the unknown parameters with the maximum likelihood method. This procedure can run into convergence troubles with small data size leading to the divergence of the model. We can resolve it by using
other alternatives. The two that we have presented in our paper are the bayesian approach and the penalized likelihood estimation.

To illustrate our "zero" option model, we undertake an application to the Video on Demand (VoD) websites. We analyse our specification in the individual and aggregate case. The aggregate model is estimated with the maximum likelihood. The same approach is not possible with the individual case because of the small data size leading to infinite estimates of the model. To solve it we use the bayesian approach. The results are satisfactory from a statistical and economic point of view. It shows a strong influence of the website composition in terms of movies and series, the novelties level as well as the hiring cost. For the tariff, the free video with advert prevail over the paying or subscription option. The consumers also prefer a VoD website composition with 75% Movies and 25% Series. They do not like websites with only old programmes. They prefer a mix with the new and old videos with a slight preference for the composition with a maximum of novelties. The services like the trailer availability, the programmes quantity and the hire length are not significant.

As for the refusal and the conflict, our application shows that the parameters associated with the no choice are significant. Therefore, the conflict and the refusal addition is really important because of the supplemental information it conveys about the individual behavior in the decision making process.

According to our empirical example we demonstrate that the bayesian approach in the dichotomous case offers better results from a statistical and economic point of view than the widespread penalized likelihood estimation. The latter that is recommended in the literature for the dichotomous logit model with data separation does not have the same effect with our probit specification. Therefore, our decision to use a bayesian approach has been conclusive and demonstrates that the penalized likelihood is not adapted to every model.

We also calculate the market shares of two existing French VoD websites, Canalplay and TF1-Vision, that are compared to one that is created in such a way that it reflects the consumer preferences. In a market with those three websites, we observe that the existing ones are in the refusal area. This means that they are not appreciated by the consumers. Additionally, had they been selected, they are too similar and lead to uncertainty of choice about these products. The introduction of the conflict has therefore an advantage as illustrated in this example in that it enables competitors whose offer is similar, to come up with this little something that will make the difference. For example, if one of the websites offers a gift or an additional free service, it could orientate the consumer towards its VoD website. We also test the market with only the existing French websites, Canalplay and TF1Vision. The market shares show a preference for TF1Vision.

In conclusion, we have specified a "zero" option model that includes the refusal and the conflict and have demonstrated the importance to insert the conflict in the decision making process model. An interesting analysis that has not been conducted in this work is to compare our specification to others like the pair comparison model without the refusal and the conflict, or the pair comparison model with the refusal only and vice versa. From an econometric and coding point of view, other perspectives could be implemented, like a logit specification of our "zero" option model that could be estimated with the bayesian and the penalized likelihood approaches to define which one gives better results. With this end in view, the maximum penalized likelihood should be available in a commercial software. The programming of this procedure could be in fact very interesting for the users.

The extension of our "zero" option model to a triple comparison of the products with the inser-
tion of the refusal and the conflict could also be of interest. However, the graphical construction
would be more complex as well as the development of the probabilities associated with the five
alternatives proposed in the questionnaire.
where \( d \) correspond to the refusal threshold denoted in the text \( \delta \), \( d_1 \) to the conflict threshold denoted in the text \( \delta' \) and \( b_1 - b_{10} \) to the parameter \( \beta_1 - \beta_{10} \) the parameters associated with each level of each attribute.
Appendix 2

Table A1
RESULTS FROM THE BAYESIAN APPROACH OF THE MODEL WITH ONLY TWO OF THE FOUR ALTERNATIVES SELECTED (THE PRODUCT $h$ AND $l$)

<table>
<thead>
<tr>
<th>Individual 22</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable</td>
</tr>
<tr>
<td>Catalogue : 1000</td>
</tr>
<tr>
<td>Catalogue : 600</td>
</tr>
<tr>
<td>Composition : 100% Movies</td>
</tr>
<tr>
<td>Composition : 75% Movies 25% Series</td>
</tr>
<tr>
<td>Composition : 50% Movies 50% Series</td>
</tr>
<tr>
<td>Composition : 100% Series</td>
</tr>
<tr>
<td>Novelty : max</td>
</tr>
<tr>
<td>Novelty : half</td>
</tr>
<tr>
<td>Old : max</td>
</tr>
<tr>
<td>Tariff : paying</td>
</tr>
<tr>
<td>Tariff : free advert</td>
</tr>
<tr>
<td>Tariff : subscription</td>
</tr>
<tr>
<td>Hire length : 24h</td>
</tr>
<tr>
<td>Hire length : 48h</td>
</tr>
<tr>
<td>Availability : trailer</td>
</tr>
<tr>
<td>Availability : None</td>
</tr>
</tbody>
</table>
### Table A2
RESULTS FROM THE PENALIZED LIKELIHOOD APPROACH OF THE MODEL WITH ONLY TWO OF THE FOUR ALTERNATIVES SELECTED (THE PRODUCT $h$ AND $l$)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Utility</th>
<th>Standard Error</th>
<th>Importance (%) Utility Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Catalogue : 1000</td>
<td>-0.792</td>
<td>1.127</td>
<td>9.52%</td>
</tr>
<tr>
<td>Catalogue : 600</td>
<td>0.792</td>
<td>1.127</td>
<td></td>
</tr>
<tr>
<td>Composition : 100% Movies</td>
<td>-0.626</td>
<td>1.960</td>
<td>36.76%</td>
</tr>
<tr>
<td>Composition : 75% Movies 25% Series</td>
<td>-1.976</td>
<td>1.890</td>
<td></td>
</tr>
<tr>
<td>Composition : 50% Movies 50% Series</td>
<td>-1.528</td>
<td>1.754</td>
<td></td>
</tr>
<tr>
<td>Composition : 100% Series</td>
<td>4.131</td>
<td>5.605</td>
<td></td>
</tr>
<tr>
<td>Novelty : max</td>
<td>-2.172</td>
<td>1.411</td>
<td>37.07%</td>
</tr>
<tr>
<td>Novelty : half</td>
<td>-1.816</td>
<td>1.141</td>
<td></td>
</tr>
<tr>
<td>Old : max</td>
<td>3.988</td>
<td>2.553</td>
<td></td>
</tr>
<tr>
<td>Tariff : paying</td>
<td>0.334</td>
<td>1.273</td>
<td>5.42%</td>
</tr>
<tr>
<td>Tariff : free advert</td>
<td>-0.579</td>
<td>1.045</td>
<td></td>
</tr>
<tr>
<td>Tariff : subscription</td>
<td>0.245</td>
<td>2.319</td>
<td></td>
</tr>
<tr>
<td>Hire length : 24h</td>
<td>0.698</td>
<td>1.119</td>
<td>8.31%</td>
</tr>
<tr>
<td>Hire length : 48h</td>
<td>-0.698</td>
<td>1.119</td>
<td></td>
</tr>
<tr>
<td>Availability : trailer</td>
<td>0.242</td>
<td>1.092</td>
<td>2.89%</td>
</tr>
<tr>
<td>Availability : none</td>
<td>-0.242</td>
<td>1.092</td>
<td></td>
</tr>
</tbody>
</table>

Likelihood ratio test=9.925 on 10 df, $p=0.447$, $n=20$
### Appendix 3

**Table A3**  

**Individual 2**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Utility</th>
<th>Standard Error</th>
<th>Importance (% Utility Range)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Refusal : $\delta$</td>
<td>1.271</td>
<td>0.083</td>
<td></td>
</tr>
<tr>
<td>Conflict : $\delta'$</td>
<td>0.476</td>
<td>0.036</td>
<td></td>
</tr>
<tr>
<td>Catalogue : 1000</td>
<td>0.210</td>
<td>0.056</td>
<td>5.55%</td>
</tr>
<tr>
<td>Catalogue : 600</td>
<td>-0.210</td>
<td>0.056</td>
<td></td>
</tr>
<tr>
<td>Composition : 100% Movies</td>
<td>0.240</td>
<td>0.084</td>
<td>31.63%</td>
</tr>
<tr>
<td>Composition : 75% Movies 25% Series</td>
<td>0.751</td>
<td>0.097</td>
<td></td>
</tr>
<tr>
<td>Composition : 50% Movies 50% Series</td>
<td>0.654</td>
<td>0.078</td>
<td></td>
</tr>
<tr>
<td>Composition : 100% Series</td>
<td>-1.645</td>
<td>0.260</td>
<td></td>
</tr>
<tr>
<td>Novelty : max</td>
<td>0.658</td>
<td>0.067</td>
<td>25.05%</td>
</tr>
<tr>
<td>Novelty : half</td>
<td>0.581</td>
<td>0.073</td>
<td></td>
</tr>
<tr>
<td>Old : max</td>
<td>-1.240</td>
<td>0.140</td>
<td></td>
</tr>
<tr>
<td>Tariff : paying</td>
<td>-0.224</td>
<td>0.054</td>
<td>33.39%</td>
</tr>
<tr>
<td>Tariff : free advert</td>
<td>1.377</td>
<td>0.060</td>
<td></td>
</tr>
<tr>
<td>Tariff : subscription</td>
<td>-1.153</td>
<td>0.114</td>
<td></td>
</tr>
<tr>
<td>Hire length : 24h</td>
<td>-0.094</td>
<td>0.056</td>
<td>2.48%</td>
</tr>
<tr>
<td>Hire length : 48h</td>
<td>0.094</td>
<td>0.056</td>
<td></td>
</tr>
<tr>
<td>Availability : trailer</td>
<td>-0.071</td>
<td>0.058</td>
<td>1.88%</td>
</tr>
<tr>
<td>Availability : None</td>
<td>0.071</td>
<td>0.058</td>
<td></td>
</tr>
</tbody>
</table>
Appendix 4

Table A4
RESULTS FROM THE MAXIMUM LIKELIHOOD ESTIMATION OF THE AGGREGATE MODEL

Aggregate model

<table>
<thead>
<tr>
<th>Variable</th>
<th>Utility</th>
<th>Standard Error</th>
<th>Importance (% Utility Range)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Refusal : $\delta$</td>
<td>1.308</td>
<td>0.112</td>
<td></td>
</tr>
<tr>
<td>Conflict : $\delta'$</td>
<td>0.471</td>
<td>0.053</td>
<td></td>
</tr>
<tr>
<td>Catalogue : 1000</td>
<td>0.212</td>
<td>0.085</td>
<td>3.14%</td>
</tr>
<tr>
<td>Catalogue : 600</td>
<td>-0.212</td>
<td>0.085</td>
<td></td>
</tr>
<tr>
<td>Composition : 100% Movies</td>
<td>0.262</td>
<td>0.121</td>
<td>35.51%</td>
</tr>
<tr>
<td>Composition : 75% Movies 25% Series</td>
<td>0.800</td>
<td>0.128</td>
<td></td>
</tr>
<tr>
<td>Composition : 50% Movies 50% Series</td>
<td>0.676</td>
<td>0.112</td>
<td></td>
</tr>
<tr>
<td>Composition : 100% Series</td>
<td>-1.597</td>
<td>0.362</td>
<td></td>
</tr>
<tr>
<td>Novelty : max</td>
<td>0.690</td>
<td>0.090</td>
<td>29.71%</td>
</tr>
<tr>
<td>Novelty : half</td>
<td>0.625</td>
<td>0.088</td>
<td></td>
</tr>
<tr>
<td>Old : max</td>
<td>-1.315</td>
<td>0.178</td>
<td></td>
</tr>
<tr>
<td>Tariff : paying</td>
<td>-0.235</td>
<td>0.079</td>
<td>23.49%</td>
</tr>
<tr>
<td>Tariff : free advert</td>
<td>1.350</td>
<td>0.081</td>
<td></td>
</tr>
<tr>
<td>Tariff : subscription</td>
<td>-1.115</td>
<td>0.160</td>
<td></td>
</tr>
<tr>
<td>Hire length : 24h</td>
<td>-0.114</td>
<td>0.078</td>
<td>3.39%</td>
</tr>
<tr>
<td>Hire length : 48h</td>
<td>0.114</td>
<td>0.078</td>
<td></td>
</tr>
<tr>
<td>Availability : trailer</td>
<td>-0.053</td>
<td>0.082</td>
<td>1.58%</td>
</tr>
<tr>
<td>Availability : None</td>
<td>0.053</td>
<td>0.082</td>
<td></td>
</tr>
</tbody>
</table>
### Appendix 5

**Table A5**  
RESULTS FROM THE MAXIMUM LIKELIHOOD ESTIMATION OF THE AGGREGATE MODEL WITHOUT THE ATTRIBUTE REFERENCE LEVELS

CONVERGENCE ACHIEVED AFTER 19 ITERATIONS

618 FUNCTION EVALUATIONS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Standard Estimate</th>
<th>Error</th>
<th>t-statistic</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>1.308</td>
<td>0.112</td>
<td>11.638</td>
<td>[0.000]</td>
</tr>
<tr>
<td>D1</td>
<td>0.471</td>
<td>0.053</td>
<td>8.825</td>
<td>[0.000]</td>
</tr>
<tr>
<td>B1</td>
<td>0.212</td>
<td>0.085</td>
<td>2.493</td>
<td>[0.013]</td>
</tr>
<tr>
<td>B2</td>
<td>0.262</td>
<td>0.121</td>
<td>2.163</td>
<td>[0.030]</td>
</tr>
<tr>
<td>B3</td>
<td>0.800</td>
<td>0.128</td>
<td>6.239</td>
<td>[0.000]</td>
</tr>
<tr>
<td>B4</td>
<td>0.676</td>
<td>0.112</td>
<td>6.009</td>
<td>[0.000]</td>
</tr>
<tr>
<td>B5</td>
<td>0.690</td>
<td>0.090</td>
<td>7.659</td>
<td>[0.000]</td>
</tr>
<tr>
<td>B6</td>
<td>0.625</td>
<td>0.088</td>
<td>7.080</td>
<td>[0.000]</td>
</tr>
<tr>
<td>B7</td>
<td>-0.235</td>
<td>0.079</td>
<td>-2.965</td>
<td>[0.003]</td>
</tr>
<tr>
<td>B8</td>
<td>1.350</td>
<td>0.081</td>
<td>16.615</td>
<td>[0.000]</td>
</tr>
<tr>
<td>B9</td>
<td>-0.114</td>
<td>0.078</td>
<td>-1.450</td>
<td>[0.147]</td>
</tr>
<tr>
<td>B10</td>
<td>-0.053</td>
<td>0.082</td>
<td>-0.648</td>
<td>[0.516]</td>
</tr>
</tbody>
</table>

where

D    Refusal  
D1   Conflict  
B1   Catalogue : 1000  
B2   Composition : 100% Movies  
B3   Composition : 75% Movies 25% Series  
B4   Composition : 50% Movies 50% Series  
B5   Novelty : max  
B6   Novelty : half  
B7   Tariff : paying  
B8   Tariff : free advert  
B9   Hire length : 24h  
B10  Availability : trailer
## Appendix 6

### Table A6
ESTIMATES AND UTILITIES OF THE CANALPLAY WEBSITE

<table>
<thead>
<tr>
<th>Individual 2</th>
<th>Canalplay</th>
<th>Attributes</th>
<th>Levels</th>
<th>Estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Catalogue</td>
<td>1000</td>
<td>0.210</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Composition</td>
<td>100% Movies</td>
<td>0.240</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Novelty</td>
<td>half</td>
<td>0.581</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Tariff</td>
<td>paying</td>
<td>-0.224</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Hire length</td>
<td>24h</td>
<td>-0.094</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Availability</td>
<td>none</td>
<td>0.071</td>
<td></td>
</tr>
</tbody>
</table>

Utility $u_i =$ Canalplay

= Canalplay website utility

for the individual 2

Individual purchase

$$\frac{\sum u_i}{\sum \exp(u_i)}$$

(Bradley-Terry-Luce) 16.773%

probabilities in %

$$\frac{\exp(u_i)}{\sum \exp(u_i)}$$

(logit) 7.850%

### Table A7
ESTIMATES AND UTILITIES OF THE TF1VISION WEBSITE

<table>
<thead>
<tr>
<th>Individual 2</th>
<th>TF1Vision</th>
<th>Attributes</th>
<th>Levels</th>
<th>Estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Catalogue</td>
<td>600</td>
<td>-0.210</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Composition</td>
<td>75% Movies &amp; 25% Series</td>
<td>0.751</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Novelty</td>
<td>half</td>
<td>0.581</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Tariff</td>
<td>paying</td>
<td>-0.224</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Hire length</td>
<td>24h</td>
<td>-0.094</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Availability</td>
<td>trailer</td>
<td>-0.071</td>
<td></td>
</tr>
</tbody>
</table>

Utility $u_i =$ TF1Vision

= TF1Vision website utility

for the individual 2

Individual purchase

$$\frac{\sum u_i}{\sum \exp(u_i)}$$

(Bradley-Terry-Luce) 15.657%

probabilities in %

$$\frac{\exp(u_i)}{\sum \exp(u_i)}$$

(logit) 7.451%

### Table A8
ESTIMATES AND UTILITIES OF THE IDEAL WEBSITE

<table>
<thead>
<tr>
<th>Individual 2</th>
<th>Ideal</th>
<th>Attributes</th>
<th>Levels</th>
<th>Estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Catalogue</td>
<td>600</td>
<td>0.210</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Composition</td>
<td>75% Movies &amp; 25% Series</td>
<td>0.751</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Novelty</td>
<td>half</td>
<td>0.658</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Tariff</td>
<td>paying</td>
<td>1.377</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Hire length</td>
<td>24h</td>
<td>0.094</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Availability</td>
<td>trailer</td>
<td>0.071</td>
<td></td>
</tr>
</tbody>
</table>

Utility $u_i =$ Ideal

= Ideal website utility

for the individual 2

Individual purchase

$$\frac{\sum u_i}{\sum \exp(u_i)}$$

(Bradley-Terry-Luce) 67.568%

probabilities in %

$$\frac{\exp(u_i)}{\sum \exp(u_i)}$$

(logit) 84.697%
REFERENCES


Baron J. and Ritov I. (1994), "Reference Points and Omission Bias", Organizational Behavior and Human Decision Processes, 59 (September), 475-498

Beattie J. and Barlas S. (1992), "Predicting Perceived Differences in Tradeoff Difficulty", working paper, University of Sussex


Bull S. B., Mak C., and Greenwood C. M. T. (2002), "A modified score function estimator for multinomial logistic regression in small samples", Computational Statistics and Data Analysis, 39, 57-74

Congdon P. (2001), "Bayesian Statistical Modelling, Chichester, UK : John Wiley

Dagnelie P. (2003), Principe d’expérimentation : Planification des expériences et analyse de leurs résultats, Les presses agronomiques de Gembloux


Firth D. (1993), "Bias reduction of maximum likelihood estimates", Biometrika, 80, 1, pp. 27-38


Heinze G. and Ploner M. (2003), "Fixing the nonconvergence bug in logistic regression with SPLUS and SAS", Computer Methods and Programs in Biomedicine, 71, 181-187

Heinze G. and Schemper M. (2002), "A solution to the problem of separation in logistic regression", Statistics in Medicine, 21, 2409-2419


Cite this article as : Roselinde Kessels, Peter Goos, and Martina Vandebroek, "Optimal designs for conjoint experiments", Computational Statistics and Data Analysis (2007), doi : 10.1016/j.csda.2007.10.016


Lollivier S. (2006), "Econométrie avancée des variables qualitatives", Economica


Maddala G. S. (1983), Limited Dependent and Qualitative Variables in Econometrics, Cambridge University Press


Ritov I. and Baron J. (1990), "Reluctance to Vaccinate : Omission Bias and Ambiguity", Journal of Behavioral Decision Making, 3 (December), 263-277

Saporta G. (2006), "Probabilités, analyse des données et statistique", 2ème édition, Technip

Scholnick E. K. and Wing C. S. (1988), "Knowing When You Don’t Know : Developmental and Situational Considerations", Developmental Psychology, 24 (March), 190-196


Thomas A.(2000), "Econométrie des variables qualitatives", Dunod


Zorn C. (2005), "A solution to separation in binary response models", Political Analysis, Volume 13, Number 2, 157-170

The computer French magazines : L’Ordinateur Individuel, P2PMag, Micro Actuel, Micro hebdo

Some newspapers : Le Monde, Le Temps