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Interval observer-based supervision of nonlinear networked control systems

Afef Najjar[1,*], **Thach Ngoc Dinh**[2], **Messaoud Amairi**[1], **Tarek Raissi**[2]
[1]Research Laboratory Modeling, Analysis and Control of Systems (MACS) LR16ES22,
National Engineering School of Gabes (ENIG),University of Gabes,Gabes, Tunisia
[2]Conservatoire National des Arts et Métiers (CNAM), Cedric, 292 Rue St-Martin, Paris Cedex, France

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Abstract: Networked control system (NCS) is a multidisciplinary area that attracts increasing attention today. In this paper, we deal with remote supervision of a nonlinear networked control systems class subject to network imperfections. Different from many existing researches that consider only the problem of small and/or constant communication delays, we focus on large and time-varying network delays problem in both measurement and control channels. The proposed method is a set-membership estimation-based predictor approach computing a guaranteed set of admissible state values when the uncertainties (i.e. measurement noises and system disturbances) are considered unknown but bounded with a priori known bounds. The state prediction strategy is used to compensate the effect of transmission delays. Finally, the theoretical results are validated through a numerical example.

Key words: Nonlinear networked control systems, remote supervision, uncertain large time-varying delay, interval estimation, input-to-state stability, predictor

1. Introduction

In last decades, the accelerating technological development of networks introduced new challenges in different real-time applications: teleoperation system [1], unmanned vehicle [2] and intelligent transportation [3], etc. In such applications, a plant to be controlled, or simply supervised, is located far from the computing unit and the measured data are exchanged among spatially isolated system components (actuators, sensors, controllers and other nodes) via communication links. This special class of distributed systems is known as networked control systems (NCSs) whose operation is coordinated over a wide area via both wired and wireless links [4]. A NCS can be presented under one of three main configurations: centralized configuration, decentralized configuration and distributed configuration.

In case of centralized configuration, there are the plant, the centralized controller, sensors and actuators. Within this configuration, the information on each sensor node should be transmitted to the centralized controller, which ensures generally an optimal control performance. However, this structure may lead to some problems such as the total system control dependence on the centralized controller and an increased computational load in the central processing unit which cannot occur in the decentralized configuration. Different from the first structure, decentralized configuration contains local controllers instead of a centralized one. Each local controller should supervise its local devices without sharing data with other controllers nodes, which eliminates the problem of increased computational load. The main drawback of this architecture is the

*Correspondence: afef.najjar@enig.rnu.tn

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possibility of global system deterioration if the decentralized controllers are not well coordinated. Finally, if an NCS is designed within the distributed configuration, we speak about distributed networked control system. Different from decentralized configuration, there are interconnections between controllers nodes which offer more robustness and scalability. Further details of these different configurations and researches based on each one could be found in recent surveys [4, 5] and the references therein.

To connect these different nodes and avoid node collision problem, specially in multiple packet transmission scenario, protocols are necessary such as the Internet, Ethernet, Transfer Control Protocol (TCP), User Data Protocol (UDP), Controller Area Network bus (CAN), Fieldbus, Profibus, etc. Thanks to its distributed nature, NCS architectures eliminate unnecessary and expensive wiring, as well as making diagnostic, maintenance and installation more flexible and easier [4]. Despite these important advantages over traditional feedback control, many constraints can appear in NCSs, which are notably generated by the limited capacity of the communication channel or the overloading network equipment and control nodes. Delays, packets losses and sampling problems are the most common ones [4, 5].

Neglecting the effects of one or more of these communication medium might lead to performance deterioration or even system destabilization in some cases. To deal with these challenging issues, many techniques have been developed in the literature [4]. For example, NCS was treated as a Markovian system in [6] and a switched linear system model was used for NCS analysis in other contributions [7, 8]. For bounded transmission delays, the authors in [9] focus on NCS input-to-state stability (ISS) using the impulsive system approach, while in [8] linear NCS exponential stabilization problem was addressed. When network-induced delays and/or packet dropouts are random, the stochastic system approach can be applied as in [10] to achieve stabilization condition for a linear NCS subject to stochastic input delays. Furthermore, it is important to mention that a great attention was paid to the use of time-delay models, which is unavoidable in NCS analysis and design since data transmission over a digital network is not instantaneous. Thus, the closed-loop system was studied as a system with input delay and other time-varying delays [8–11]. In these works, a predictive approach applied to input delay systems has been considered. In fact, the predictive approach has been well required to compensate time delays and packet dropouts effects in communication channels. In [12, 13], a network predictive controller was designed to compensate random network delays, guaranteeing the stability of the global system. In addition, a distributed model predictive control of large-scaled system subject to network-induced delays was investigated in [14] to achieve exponential stability of the overall system.

On the other hand, system states are often required for NCS research. Nonetheless, it may not always be possible in practice to measure all of the system states. Hence, state observation techniques can be a solution to estimate unmeasured state components. For instance, conventional observers have been generally designed (see [1, 6–8, 13, 14]). However, it is worth pointing out that the estimation issues become much more challenging in the presence of important and fluctuating system uncertainties. If these uncertainties are not known, changing and large; or cannot be described in stochastic terms, the aforementioned methods are no longer applicable because classical observers by themself are not equipped with explicit guarantees in the presence of unknown deterministic perturbations. This issue motivates the present work. Here, an approach named interval observer framework has been proposed. This approach differs from the classical one by calculating a lower and upper bound for the state. This approach was introduced for the first time in [15] and has been expanded to various works for both continuous-time systems and discrete-time ones [16–21].

Much remains to be done. This paper aims to develop interval observer based monitoring strategy. As

shown in Figure 1, its typical scheme is comprised of two remote sides: the data acquisition and control side and the remote plant side linked via a network.

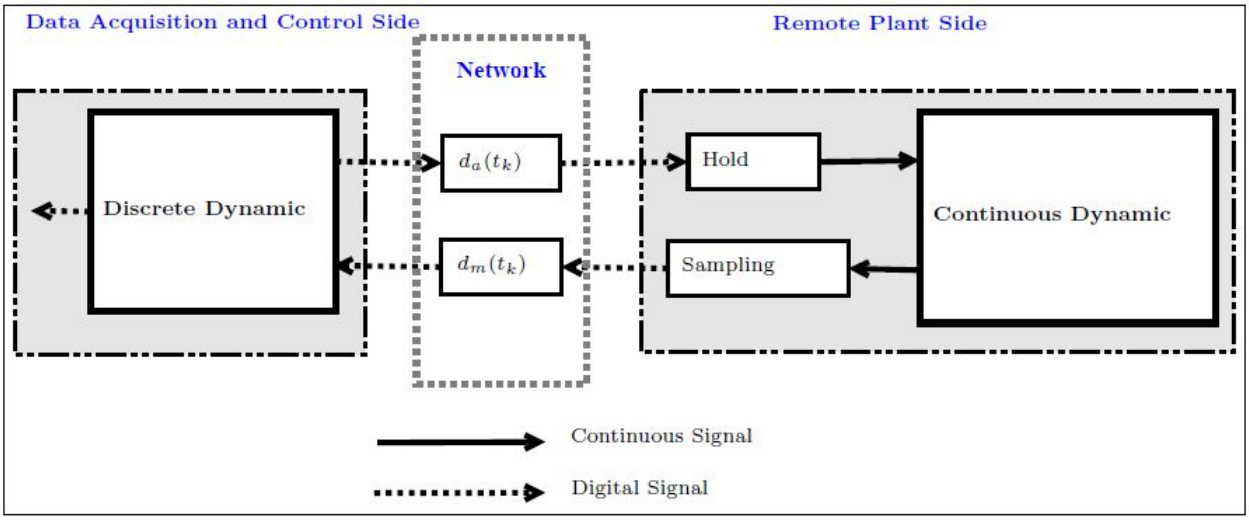


Figure 1. The typical NNCS schematic.

The data acquisition and control side supervise the remote physical system. Before the transmission process, the data pass through a sampling stage, which is done with a constant period.

We consider nonlinear networked control systems (NNCS) class subject to uncertain network delays in both measurement and control channels. Different from many works studying only the case of small network-induced delays as in [11] or constant network-induced delays [7, 13], we treat a NNCS subject to large (longer than the sampling period) and time varying delays. These delays are unknown but bounded with known bounds. First, since not all the states are available in practice, a local interval observer is proposed to calculate the plant states (see Figure 1). We ensure input-to-state stability condition for the observation errors. Then, after a finite time depending on the upper bound of transmission delay in the measurement channel, the state predictor provides upper and lower bounds for each component of the solutions with the presence of unknown but bounded uncertainties (i.e. disturbances and measurement noises) to compensate communication delays. Thanks to the state predictor outputs, the remote plant state is reconstructed. Furthermore, the contributions of this paper are different from [22] and overcome important problems in NCS studies. These modifications consist on two main important points: firstly, in [22] a local and instantaneous input signal was considered while in the present contribution, we focus on remote input signal transmission through a delayed control channel, which is often the case of real applications. Consequently, the NCS is modeled as an unknown input delay system, which makes the design of our interval observer and our predictor of state challenging. Secondly, the robustness of interval observer is discussed in our work by guaranteeing input-to-state stability of the observation errors.

The organization of this paper is as follow: Section 2 describes some useful notations and preliminaries. Section 3 presents the system under consideration. The interval observer design is developed in Section 4. Section 5 investigates the predictor technique. A numerical example is given in Section 6 to validate the theoretical developments. Finally, the paper ends with concluding remarks and some open problems.

Nomenclature:	
CAN	Controller Area Network
CT	Continuous Signal
DAS	Data Acquisition Side
DS	Digital Signal
ISS	Input-to-State Stability
NCS	Networked control systems
NNCS	Nonlinear networked control systems
RPS	Remote Plant Side
Sp	Sampling
TCP	Transfer Control Protocol
UDP	User Data Protocol

2. Preliminaries, definition and notations

Throughout this paper, the symbols $\mathbb{R}$ and $\mathbb{N}$ stand for the sets of real and natural numbers, respectively. The eigenvalues set of a matrix $A \in \mathbb{R}^{n\times n}$ is named $\lambda(A)$ and $\mathrm{Re}(z)$ represents the real part of the complex number z. The set of Hurwitz matrices from the set $\mathbb{R}^{n\times n}$ is denoted by $\mathbb{H}$, i.e. $R \in \mathbb{H} \Leftrightarrow \mathrm{Re}(\lambda) < 0, \forall \lambda \in \lambda(R)$. We denote $\mathbb{M}$ as the set of Metzler matrices from the set $\mathbb{R}^{n\times n}$, i.e. $R = \{r_{ij}\}_{i,j=1}^{n} \in \mathbb{M} \Leftrightarrow r_{i,j} \geq 0$ for $i \neq j$. For a variable $x(t) \in \mathbb{R}^n$, the upper and lower bounds are denoted by $\overline{x}(t) \in \mathbb{R}^n$ and $\underline{x}(t) \in \mathbb{R}^n$, respectively, such that $\underline{x}(t) \leq x(t) \leq \overline{x}(t)$ and the relation $\leq$ should be interpreted elementwise for vectors and matrices, i.e. $A = (a_{i,j}) \in \mathbb{R}^{n\times m}$ and $B = (b_{i,j}) \in \mathbb{R}^{n\times m}$ such that $A \geq B$ if and only if $a_{i,j} \geq b_{i,j}$ for all $i \in \{1, ..., n\}$ and $j \in \{1, ..., m\}, i, j \in \mathbb{N}$.

For a matrix $R \in \mathbb{R}^{n\times m}$, define $R^+ = \max\{0, R\}$ and $R^- = R^+ - R$. The matrix of absolute values of all elements of a matrix $M \in \mathbb{R}^{n\times m}$ is $|M| = M^+ + M^-$. The vector E_p is stated for $(p \times 1)$ vector with unit elements, and I_n denotes the identity matrix of $n \times n$ dimension. Superscript T denotes the transpose of a matrix or a vector. The set $\mathcal{K}$ is the set of continuous increasing functions $\gamma : \mathbb{R}_+ \to \mathbb{R}_+$ with $\gamma(0) = 0$. Also we denote $\beta \in \mathcal{KL}$ if $\beta(\cdot, t) \in \mathcal{K}$ for all $t \geq 0$ and $\beta(r, \cdot)$ is continuous and strictly decreasing to zero for all $r > 0$. $\|.\|$ refers to the standard 2-norm.

Lemma 1 *[16] Let $\underline{x}, x, \overline{x} \in \mathbb{R}^n$ be vectors satisfying $\underline{x} \leq x \leq \overline{x}$ and $A \in \mathbb{R}^{n\times m}$ be a time-invariant matrix. Then, the inequalities below hold:*

$$A^+\underline{x} - A^-\overline{x} \leq Ax \leq A^+\overline{x} - A^-\underline{x}. \tag{1}$$

□

Definition 1 *[15] A continuous-time linear system defined by $\dot{x}(t) = Ax(t)$ is called cooperative, positive or nonnegative if $A \in \mathbb{M}$.* □

Lemma 2 *[15] Consider the following system:*

$$\begin{cases} \dot{x}(t) = Ax(t) + \psi(t), \\ y(t) = Cx(t), \end{cases} \tag{2}$$

where $\psi(t)$ is a continuous function and A, C are known matrices. Suppose that there exist two known continuous-time functions $\underline{\psi}(t)$ and $\overline{\psi}(t) : \mathbb{R} \to \mathbb{R}^n$ satisfied $\underline{\psi}(t) \leq \psi(t) \leq \overline{\psi}(t)$, for all $t \geq 0$.

If there exists a gain L *such that* $(A-LC) \in \mathbb{H} \cap \mathbb{M}$ *and* $\underline{x_0}, x_0, \overline{x_0} \in \mathbb{R}^n$, $\underline{x}_0 \leq x_0 \leq \overline{x}_0$, *then the system:*

$$\begin{cases} \dot{\overline{x}}(t) = A\overline{x}(t) + \overline{\psi}(t) + L(y(t) - C\overline{x}(t)) \\ \dot{\underline{x}}(t) = A\underline{x}(t) + \underline{\psi}(t) + L(y(t) - C\underline{x}(t)) \end{cases} \tag{3}$$

is an interval observer for (2) and $\underline{x}(t) \leq x(t) \leq \overline{x}(t)$, $\forall t \geq 0$. □

Definition 2 *[23] Consider the nonlinear system*

$$\dot{x} = f(x, u), \tag{4}$$

with $f(x,u) \in \mathbb{R}^n$, *the system (4) is input-to-state Stable (ISS) if for any input* $u \in \mathbb{R}^m$ *and* $x_0 \in \mathbb{R}^n$ *there exist functions* $\beta \in \mathcal{KL}$ *and* $\gamma \in \mathcal{K}$ *such that*

$$|x(t, x_0, u)| \leq \beta(x_0, t) + \gamma(\|u\|), \quad \forall t \geq 0. \tag{5}$$

□

3. NCS description and problem formulation

In this work, we aim to supervise a remote nonlinear physical plant described by:

$$\begin{cases} \dot{x}(t) = Ax(t) + F(u(t), y(t)) + w(t), \\ y(t) = Cx(t) + v(t), \end{cases} \tag{6}$$

where $x \in \mathbb{R}^n$, $y \in \mathbb{R}^p$ and $u \in \mathbb{R}^m$ represent respectively the state, the output and the inputs of the physical plant, where $w \in \mathbb{R}^n$ and $v \in \mathbb{R}^p$ are the external disturbances and the measurement noises, respectively. The functions u, v, w are continuous. The function $F(u(t), y(t)) \in \mathbb{R}^n$ is a globally Lipschitz nonlinear function. The matrices A and C are two known constant matrices with compatible dimensions.

Before proceeding further, we make some assumptions on the physical plant matrices.

Assumption 1 *The pair* (A, C) *is detectable.*

Assumption 2 *The additive disturbances and the measurement noise are unknown but bounded with a priori known bounds:*

$$\underline{w} \leq w(t) \leq \overline{w}, \quad |v(t)| \leq \overline{V} E_p, \quad \forall t \geq 0 \tag{7}$$

where $\overline{w}$, $\underline{w} \in \mathbb{R}^n$, $\overline{V} \in \mathbb{R}_+$ *and* E_p *is a* $(p \times 1)$ *vector whose elements are equal to 1 (see Section 2).*

Discussions of the assumptions: Assumption 1 is required for remote physical plant state estimation. It means that there exists a constant matrix L such that the matrix $A - LC$ is Hurwitz, while Assumption 2 means that the measurement noises and disturbances are unknown but bounded. Based on these assumptions, we will design the interval observer in the following section.

The network induces two uncertain delays: $d_a(t)$ and $d_m(t)$ which refer to the actuation and the measurement channels delay, respectively. These delay functions are unknown but bounded:

$$0 \leq \underline{d_a} \leq d_a(t) \leq \overline{d_a}, \quad 0 \leq \underline{d_m} \leq d_m(t) \leq \overline{d_m}, \quad \forall t, \tag{8}$$

where $\overline{d_a}$, $\underline{d_a}$ are respectively the upper and lower bounds of $d_a(t)$ and $\overline{d_m}$, $\underline{d_m}$ are respectively the upper and lower bounds of $d_m(t)$.

Suppose that $\overline{d}$ and $\underline{d}$ are respectively the maximum and the minimum transmission delays which may occur over network links such that:

$$\overline{d} = \max\{\overline{d_a}, \overline{d_m}\}, \underline{d} = \min\{\underline{d_a}, \underline{d_m}\}. \tag{9}$$

Considering the network-induced time delay in the actuation channel, the plant (6) can be modeled as input delayed system:

$$\begin{cases} \dot{x}(t) = Ax(t) + F(u(t - d_a(t)), y(t)) + w(t) \\ y(t) = Cx(t) + v(t). \end{cases} \tag{10}$$

Assumption 3 *We assume that $u(t - d_a(t))$ is bounded.*

Let $\{t_k, k \in \mathbb{N}\}$ be the sequence of sampling instants such that $t_{k+1} - t_k = T$ and $\lim_{k\to\infty} t_k = \infty$, T is the sampling period and t_k is an increasing sequence such that $t_k = kT$.

Before its transmission over the network, the data must be sampled. Thus the following assumptions are considered.

Assumption 4 *[12] The sampling network-induced delays in control and measurement channel $d_a(t_k)$ and $d_m(t_k)$ respectively, are unknown but bounded with a priori known bounds and the upper bound $\overline{d}$ is associated to be a multiple of the sampling period T.*

Assumption 5 *[12] For any discrete-time instant $t_k \geq \overline{d}$, the information on the control signal $u(t_k)$, the information on the output $y(t_k)$ and the information on the sampling upper and lower bounds $\underline{z}(t_k)$, $\overline{z}(t_k)$ could be stored and used for all $t_k \in \left[t_k - \overline{d}, t_k\right)$.*

Note that Assumption 4 is related to the real case and can be frequently satisfied in practice. Assumption 5 is necessary for the predictor design.

As shown in Figure 2, to supervise remotely a NNCS described by the system (10) despite network constraints the following scenario is proposed:

- The input set point generation (I.S.P.G) delivers a digital input signal $u(t_k)$ which is packed and sent via a communication link (actuator channel) to the remote plant side.

- The actuator node receives $u(t - d_a(t))$ through the Hold (zero order hold for example) and applies it on the process inputs.

- Based on the measured process output $y(t)$ and the measured process input $u(t - d_a(t))$, the interval observer gives a guaranteed upper and lower bounds enclosing the system states ($\overline{x}(t)$, $\underline{x}(t)$).

- After being sampled with the fixed period T, the estimated state vector is packed and transmitted to the data acquisition side through another link (measurement channel).

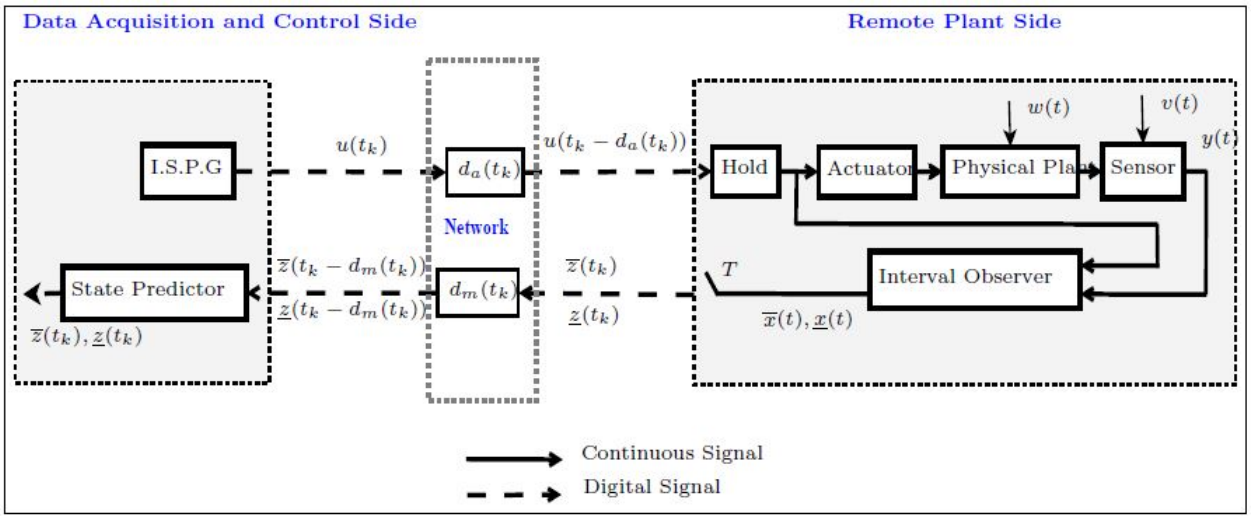


Figure 2. The interval estimator-based predictor schematic.

- A state predictor implemented in a calculator reconstructs the discrete upper and lower bounds of the states ($\overline{z}(\frac{t_k}{T})$ and $\underline{z}(\frac{t_k}{T})$) after a finite time $t_k = \overline{d}$ using the upper-delayed deviations $\overline{z}(\frac{t_k}{T} - \frac{\overline{d}}{T})$, $\underline{z}(\frac{t_k}{T} - \frac{\overline{d}}{T})$.

Remark 1 *For the sake of simplicity, the network links are supposed to be noiseless, and there are no data loses. This assumption is rather restrictive. Packet dropouts are random, time varying and no rules to follow. It is not obvious to deal with it together with random time delays in the context of interval observer/predictor design. We will consider package dropout as a future work.*

4. Interval observer design

While classical estimation can be established only when the dynamics of process disturbances $w(t)$ and the measurement noises $v(t)$ are known a priori, interval estimation requires only the bounds of these uncertainties to compute the guaranteed set of admissible values (interval). In this section, an interval observer design is tackled to estimate the unavailable states of the physical plant (10) under unknown input delay $d_a(t)$ and uncertainties (i.e the disturbances $w(t)$ and the measurement noises $v(t)$). Since it is not always possible to compute a gain L for the system (10) such that $A - LC$ is both Metzler and Hurwitz, a change of coordinates $\xi = Sx$ with a nonsingular matrix S such that the matrix $S(A - LC)S^{-1} \in \mathbb{H} \cap \mathbb{M}$ [18] is used to relax this restriction.

Theorem 1 *Let Assumptions 1–3 be satisfied and $\underline{x_0} \leq x_0 \leq \overline{x_0}$. If there exists a change of coordinates $\xi = Sx$ satisfying $\mathcal{E} = S(A - LC)P \in \mathbb{H} \cap \mathbb{M}$, $P = S^{-1}$ so that the following system*

$$\begin{cases} \dot{\hat{x}}^+(t) = \mathcal{E}\hat{x}^+(t) + \overline{SF}(u(t-\underline{d}), y(t)) + SLy(t) + S^+\overline{w} - S^-\underline{w} + |SL|\, E_p\overline{V}, \\ \dot{\hat{x}}^-(t) = \mathcal{E}\hat{x}^-(t) + \underline{SF}(u(t-\underline{d}), y(t)) + SLy(t) + S^+\underline{w} - S^-\overline{w} - |SL|\, E_p\overline{V}, \end{cases} \tag{11}$$

where

$$\begin{cases} \overline{SF}(u(t-\underline{d}), y(t)) = \max\limits_{\alpha \in [0, \overline{d}-\underline{d}]} \{SF(u(t-\underline{d}-\alpha), y(t))\}, \\ \underline{SF}(u(t-\underline{d}), y(t)) = \min\limits_{\alpha \in [0, \overline{d}-\underline{d}]} \{SF(u(t-\underline{d}-\alpha), y(t))\}, \end{cases} \tag{12}$$

and the initial conditions are selected as follows:

$$\hat{x}^+(0) = S^+\overline{x}_0 - S^-\underline{x}_0, \quad \hat{x}^-(0) = S^+\underline{x}_0 - S^-\overline{x}_0, \tag{13}$$

is input-to-state stable (ISS) interval observer for the system (10) satisfying

$$\hat{x}^-(t) \leq \xi(t) \leq \hat{x}^+(t), \quad \forall t \geq 0 \tag{14}$$

where the bounds of the solution $x(t)$ are:

$$\begin{cases} \overline{x}(t) = P^+\hat{x}^+(t) - P^-\hat{x}^-(t) \\ \underline{x}(t) = P^+\hat{x}^-(t) - P^-\hat{x}^+(t) \end{cases} \tag{15}$$

such that

$$\underline{x}(t) \leq x(t) \leq \overline{x}(t), \quad \forall t \geq 0. \tag{16}$$

□

Proof 1 *First, let us start by demonstrating that the system (11) is a framer for the plant (10) guaranteeing that inclusion (16) holds $\forall t \geq 0$. Denote by $\overline{e}(t) = \hat{x}^+(t) - \xi(t)$ the upper observation error and $\underline{e}(t) = \xi(t) - \hat{x}^-(t)$ the lower observation error, we aim to show that $\overline{e}(t) \geq 0$ and $\underline{e}(t) \geq 0$, $\forall t \geq 0$.*

From (10) and (11) the dynamics of the interval estimation errors are defined as:

$$\begin{aligned} \dot{\overline{e}}(t) &= \mathcal{E}\overline{e}(t) + \overline{SF}(u(t-\underline{d}), y(t)) \\ &- SF(u(t-d_a(t)), y(t)) + \overline{\Pi}(t), \end{aligned} \tag{17}$$

$$\begin{aligned} \dot{\underline{e}}(t) &= \mathcal{E}\underline{e}(t) + SF(u(t-d_a(t)), y(t)) \\ &- \underline{SF}(u(t-\underline{d}), y(t)) + \underline{\Pi}(t), \end{aligned} \tag{18}$$

where

$$\begin{cases} \overline{\Pi}(t) = S^+\overline{w} - S^-\underline{w} - Sw(t) + SLv(t) + |SL|\, E_p\overline{V}, \\ \underline{\Pi}(t) = S^+\underline{w} - S^-\overline{w} + Sw(t) - SLv(t) + |SL|\, E_p\overline{V}. \end{cases} \tag{19}$$

According to Assumption 2, we have $\overline{\Pi}(t) \geq 0$ and $\underline{\Pi}(t) \geq 0, \forall t \geq 0$. Furthermore, the function $F(u(t), y(t))$ is globally Lipshitz, so $SF(u(t-d_a(t)), y(t))$ is defined for all $t \geq 0$ because u is continuous. By the definition in (12), $\overline{SF}(u(t-\underline{d}), y(t))$ and $\underline{SF}(u(t-\underline{d}), y(t))$ are also defined for all $t \geq 0$.

Additionally, $\mathcal{E} \in \mathbb{M}$, $\overline{SF}(u(t-\underline{d}), y(t)) - SF(u(t-d_a(t)), y(t))$ and $SF(u(t-d_a(t)), y(t)) - \underline{SF}(u(t-\underline{d}), y(t))$ are positive then, the system (17) is positive. Therefore, the inequalities $\underline{e}(0) \geq 0$, $\overline{e}(0) \geq 0$ imply that the errors $\underline{e}(t)$ and $\overline{e}(t)$ stay nonnegative for all $t \geq 0$. So, (11) is a framer for (10).

Next, we will prove the Input to State Stability of the observer (11). Let us start with the upper bound error (17), by integrating this system between 0 and t, we obtain

$$\begin{aligned} \overline{e}(t) &= e^{\mathcal{E}t}\overline{e}(0) + \int_0^t e^{\mathcal{E}(t-s)}(\overline{SF}(u(s-\underline{d}), y(s)) - SF(u(s-d_a(s)), y(s)))ds \\ &+ \int_0^t e^{\mathcal{E}(t-s)}\overline{\Pi}(s)\,\mathrm{d}s. \end{aligned} \tag{20}$$

We have $\mathcal{E} \in \mathbb{H}$, *thus there exist* $a > 0$ *and* $b > 0$ *such that*

$$\left|e^{\mathcal{E}t}\right| \leq ae^{-bt}. \tag{21}$$

Then, based on Equation (20) we get

$$\begin{aligned} |\overline{e}(t)| &\leq ae^{-bt}\,|\overline{e}(0)| \\ &+ a\int_0^t e^{-b(t-s)}(\overline{SF}(u(s-\underline{d}),y(s)) - SF(u(s-d_a(s)),y(s)))ds \\ &+ a\int_0^t e^{-b(t-s)}\overline{\Pi}(s)ds \end{aligned} \tag{22}$$

$$\begin{aligned} |\overline{e}(t)| &\leq ae^{-bt}\,|\overline{e}(0)| \\ &+ ae^{-bt}(\int_0^t e^{bs}(\overline{SF}(u(s-\underline{d}),y(s)) - SF(u(s-d_a(s)),y(s)))ds \\ &+ \int_0^t e^{bs}\overline{\Pi}(s)ds). \end{aligned} \tag{23}$$

We have $\overline{SF}(u(t-\underline{d}),y(t)) - SF(u(t-d_a(t)),y(t))$ *and* $\overline{\Pi}$ *are bounded* $\forall t \geq 0$ *then, according to Definition 2, the ISS condition of stability is guaranteed for the upper observer bound.*

Similar to (17), the integration of Equation (18) between 0 *and* t *yields the next equation:*

$$\begin{aligned} \underline{e}(t) &= e^{\mathcal{E}t}\underline{e}(0) \\ &+ \int_0^t e^{\mathcal{E}(t-s)}(SF(u(s-d_a(s)),y(s)) - \underline{SF}(u(s-\underline{d}),y(s)))ds \\ &+ \int_0^t e^{\mathcal{E}(t-s)}\underline{\Pi}(s)\,\mathrm{d}s. \end{aligned} \tag{24}$$

Using property (21) and based on Equation (24) we have

$$\begin{aligned} |\underline{e}(t)| &\leq ae^{-bt}\,|\underline{e}(0)| \\ &+ ae^{-bt}(\int_0^t e^{bs}(SF(u(s-d_a(s)),y(s)) - \underline{SF}(u(s-\underline{d}),y(s))ds \\ &+ \int_0^t e^{bs}\overline{\Pi}(s)ds). \end{aligned} \tag{25}$$

Since $SF(u(t-d_a(t)),y(t)) - \underline{SF}(u(t-\underline{d}),y(t))$ *and* $\underline{\Pi}$ *are bounded* $\forall t \geq 0$ *then, according to Definition 2, the ISS condition of stability is guaranteed also for the lower state observation error.*

From (23) and (25), we conclude that (11) is an interval estimator for (10). By consequence, since the transformation of coordinates matrix S *is bounded, it follows that (11) is a ISS interval estimator for the process (10).* □

5. State predictor design

In this part, we introduce the proposed predictor as shown in Figure 2. After being estimated by the interval observer (15) in the previous section, the observed states $\overline{x}(t)$ and $\underline{x}(t)$ will be sent via the delayed network to the second unit (see Figure 2). Thus, the second unit receives delayed observed states instead of the current ones. Therefore, the real-time supervision goal cannot be archived in this case.

As a solution, a prediction technique is proposed in this part to compensate this network negative imperfection (i.e. the large unknown network induced delay $d_m(t_k)$). The main idea of this state predictor is similar to the one used in [13] and as described by the time scenario presented in Section 3. Based on the delayed data $\overline{z}(\frac{t_k}{T}-\frac{\overline{d}}{T})$, $\underline{z}(\frac{t_k}{T}-\frac{\overline{d}}{T})$, we will reconstruct $\overline{z}(\frac{t_k}{T})$, $\underline{z}(\frac{t_k}{T})$ after a finite time $t_k=\overline{d}$.

Assumption 6 *The sampling period T is selected such that the matrix $\Phi=I_n+T\mathcal{E}$ is positive.*

Theorem 2 *If Assumptions (4)–(6) hold for all $t_k\geq\overline{d}$, thus we get:*

$$\begin{aligned}\hat{z}^+(k) &= \Phi^{k_2}\hat{z}^+(k-k_2)\\ &+ \sum_{j=1}^{k_2}\Phi^{k_2-j}T\overline{SF}(u(k-k_1-k_2+j-1),y(k-k_2+j-1))\\ &+ \sum_{j=1}^{k_2}\Phi^{k_2-j}L_1y(k-k_2+j-1)+\sum_{j=1}^{k_2}\Phi^{k_2-j}\overline{\beta},\end{aligned}\tag{26}$$

$$\begin{aligned}\hat{z}^-(k) &= \Phi^{k_2}\hat{z}^-(k-k_2)\\ &+ \sum_{j=1}^{k_2}\Phi^{k_2-j}T\underline{SF}(u(k-k_1-k_2+j-1),y(k-k_2+j-1))\\ &+ \sum_{j=1}^{k_2}\Phi^{k_2-j}L_1y(k-k_2+j-1)+\sum_{j=1}^{k_2}\Phi^{k_2-j}\underline{\beta},\end{aligned}\tag{27}$$

where $j=1,2,3,...,k_2$, $k=\frac{t_k}{T}$, $k_1=\left\lfloor\frac{d}{T}\right\rfloor$ and $k_2=\frac{\overline{d}}{T}$, $\Phi=(I_n+T\mathcal{E})$, $L_1=TSL$, $\overline{\beta}=T(S^+\overline{w}-S^-\underline{w}+|SL|\,E_p\overline{V})$, $\underline{\beta}=T(S^+\underline{w}-S^-\overline{w}-|SL|\,E_p\overline{V})$ and

$$\begin{cases}\overline{z}(k)=S^+\hat{z}^+(k)-S^-\hat{z}^-(k)\\ \underline{z}(k)=S^+\hat{z}^-(k)-S^-\hat{z}^+(k)\end{cases}\tag{28}$$

are a predictor from the sampling instant of time $k-k_2$ to k for the physical plant (10) i.e. $\overline{z}\to\overline{x}$ and $\underline{z}\to\underline{x}$ for all sampling instant $t_k\geq\overline{d}$ and the following inclusion holds

$$\underline{z}(\frac{t_k}{T})\leq x(\frac{t_k}{T})\leq\overline{z}(\frac{t_k}{T}),\quad t_k\geq\overline{d}.\tag{29}$$

□

Proof 2 *Due that sampling stage is essential before data transmission through digital links, the interval observer (11) should be rewritten under its discrete-time form. For this purpose, an Euler approximation with the sampling period T is used as below:*

$$\begin{cases} \hat{z}^+(k+1) = \Phi\hat{z}^+(k) + T\overline{SF}(u(k-k_1), y(k)) + L_1 y(k) + \overline{\beta}, \\ \hat{z}^-(k+1) = \Phi\hat{z}^-(k) + T\underline{SF}(u(k-k_1), y(k)) + L_1 y(k) + \underline{\beta}. \end{cases} \tag{30}$$

Starting by the upper predictor, from (30) and based on the information received at $k-k_2$, the state prediction from $k-k_2+1$ to k are computed successively as

$$\begin{aligned} \hat{z}^+(k-k_2+1) &= \Phi\hat{z}^+(k-k_2) \\ &+ T\overline{SF}(u(k-k_1-k_2), y(k-k_2)) \\ &+ L_1 y(k-k_2) + \overline{\beta} \end{aligned} \tag{31}$$

$$\begin{aligned} \hat{z}^+(k-k_2+2) &= \Phi\hat{z}^+(k-k_2+1) \\ &+ T\overline{SF}(u(k-k_1-k_2+1), y(k-k_2+1)) \\ &+ L_1 y(k-k_2+1) + \overline{\beta} \end{aligned} \tag{32}$$

$$\vdots$$

$$\begin{aligned} \hat{z}^+(k) &= \Phi\hat{z}^+(k-1) \\ &+ T\overline{SF}(u(k-k_1-1), y(k-1)) \\ &+ L_1 y(k-1) + \overline{\beta} \end{aligned} \tag{33}$$

From these previous equalities as in [13], we obtain (26). Following the same steps as in the upper predictor, the lower predictor $\hat{z}^-(k)$ as in (27). Then (26) is a predictor from $k-k_2$ to k of the upper bound of interval observer (11) in the new basis ξ; $\hat{z}^+ \to \hat{x}^+$ after a fixed time $t_k = \overline{d}$. Similar to the lower bound of the interval observer (11), we get $\hat{z}^- \to \hat{x}^-$.

Consequently, after the change of coordinates S, we can confirm that $\overline{z} \to \overline{x}$ and $\underline{z} \to \underline{x}$. As a result, the proposed predictor compensates the delay $d_m(k)$ for all $t_k \geq \overline{d}$. Then, the inclusion (29) is explained as follows. The change of coordinate in the previous section ensures the cooperativity of the observer error i.e. $\mathcal{E}$ is Metzler, by consequence Φ is also a Metzler matrix. Therefore, there exists T such that the matrix Φ is positive for all $t_k \geq \overline{d}$ and the inclusion (29) holds. Finally, the current observed states are regenerated and the designed system (28) is a predictor for the remote physical plant (10). □

6. Numerical exemple

To prove the efficiency of the proposed method, let us consider a nonlinear plant of the form (6) where

$$A = \begin{pmatrix} -3.5000 & 0 & 0.5000 \\ 0 & -2.7540 & 0 \\ 0 & 0 & -1.2000 \end{pmatrix}, \; F(u(t), y(t)) = \begin{pmatrix} \sin(u(t)) \\ 1.5\sin(u(t)y(t)) \\ 2\sin(u(t)) \end{pmatrix}, \; C = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix},$$

$$w(t) = [0.1\cos(2t) \quad 0.1sin(3t) \quad 0.1\cos(4t)]^T,\ \overline{w} = [0.1 \quad 0.1 \quad 0.1]^T,\ \underline{w} = -\overline{w} \text{ and}$$
$$v(t) = 0.2\cos(t)\cos(5t)\sin(10t)\sin(20t) \text{ with } \overline{V} = 0.2.$$

It is clear that the function $F(u(t), y(t))$ is globally Lipschitz. Since it is supervised via a network inducing uncertain delays, the system (6) will be described as (10) and our objective is to supervise remotely this latter via the network.

The input signal delivered by the I.S.P.G and the output signal of the system (10) are depicted in Figure 3.

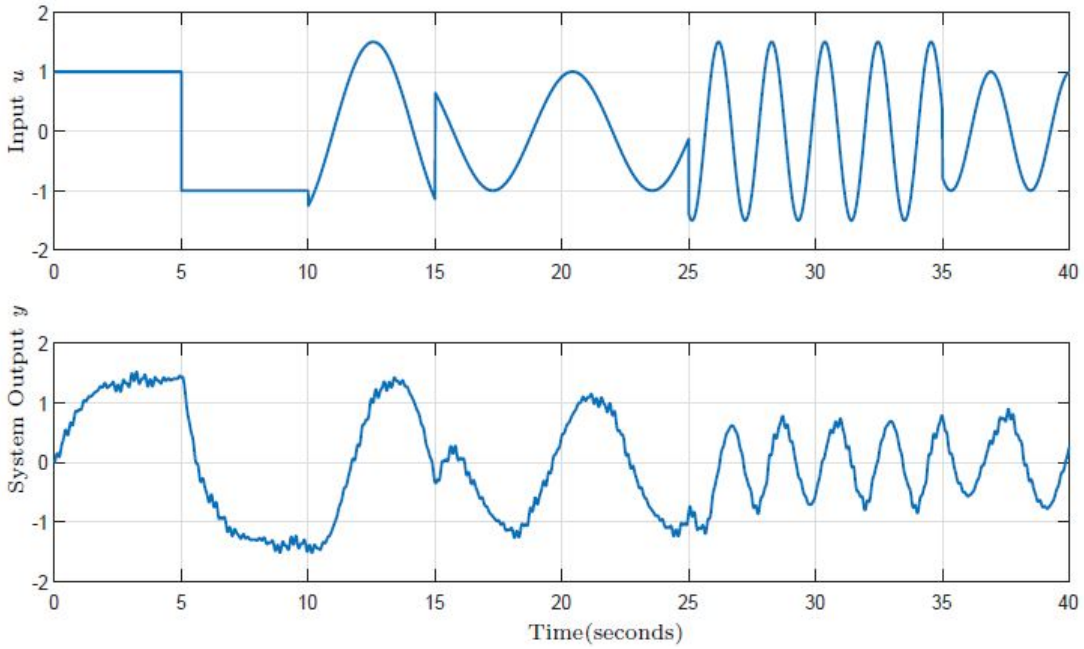


Figure 3. System input and output.

Network proprieties: The network induces uncertain but bounded delays as described by (8) in the forward and backward channels. These bounds are $\underline{d} = 0.05s$, $\overline{d} = 0.1s$ and the distribution of delays is shown in Figure 4 with the disturbances and the measurement noise. The sampling period is given by $T = 0.01s$.

Interval observer design: To design a local interval estimator for the plant states, a gain L is computed satisfying $A - LC \in \mathbb{H}$ i.e. $L = \begin{pmatrix} -10 & 0 & 0 \end{pmatrix}^T$. However, the matrix $A - LC$ is Hurwitz but is not Metzler. Then, a transformation of coordinates as described in Section 4 is needed. We propose $S = \begin{pmatrix} 0.5005 & 0.0000 & -2.2800 \\ 0.000 & 1.0064 & 0.0000 \\ -0.0010 & -0.0012 & 5.5701 \end{pmatrix}$, and we can easily verify that

$$\mathcal{E} = S(A - LC)S^{-1} = \begin{pmatrix} -3.5000 & 0.0000 & 0.0020 \\ 0.0000 & -2.7540 & 0.0000 \\ 0.0049 & 0.0019 & -1.2000 \end{pmatrix} \in \mathbb{H} \cap \mathbb{M}.$$

Therefore, an interval observer as (11) can be designed. The initial conditions of system (10) are chosen such that $\underline{x}_0(t) \leq x_0(t) \leq \overline{x}_0(t)$. We take in this example $\overline{x}(0) = \begin{pmatrix} 0.5 & 0.5 & 0.5 \end{pmatrix}^T$ and $\underline{x}(0) = \begin{pmatrix} -0.5 & -0.5 & -0.5 \end{pmatrix}^T$. Also the estimator (11) is initialized by $\hat{x}^+(0)$ and $\hat{x}^-(0)$ which are defined in (13).

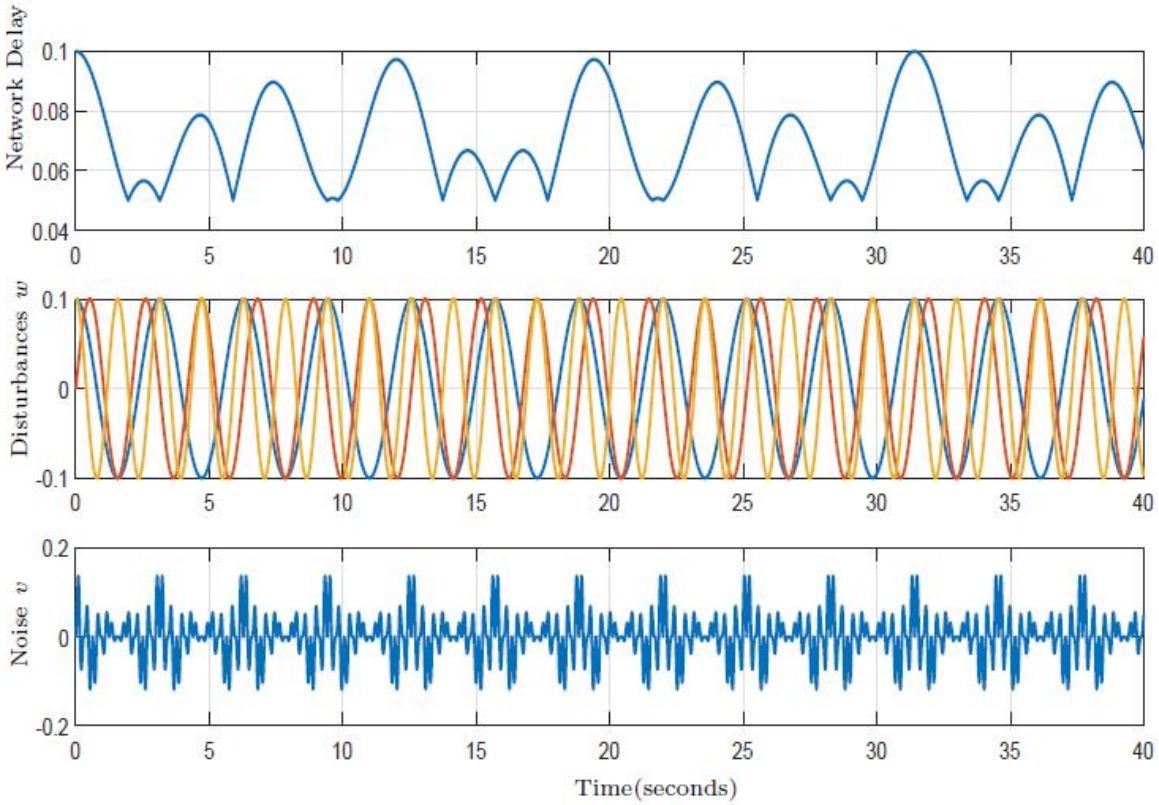


Figure 4. Network delay, disturbances and noise.

State predictor design: The sampling period T should satisfy the positivity of the matrix Φ (Assumption 6). In our case, we choose $\mathrm{T} = 0.01\mathrm{s}$. Then, $\Phi = \begin{pmatrix} 0.9650 & 0.0000 & 0.0000 \\ 0.0000 & 0.9724 & 0.0000 \\ 0.00005 & 0.00002 & 0.9880 \end{pmatrix}$ is a positive matrix and Assumption 6 is satisfied. In addition, the state predictor described by (26) and (27) can be computed with $\Phi = \begin{pmatrix} 0.9650 & 0.0000 & 0.0000 \\ 0.0000 & 0.9724 & 0.0000 \\ 0.00005 & 0.00002 & 0.9880 \end{pmatrix}$, $\tilde{L_1} = \begin{pmatrix} -0.0500 \\ 0.0000 \\ 0.0001 \end{pmatrix}$ and $\overline{\beta} = \begin{pmatrix} 0.0127 \\ 0.0010 \\ 0.00559 \end{pmatrix}$, $\underline{\beta} = -\overline{\beta}$. The actual state x and observed state are plotted in Figure 5, where the continuous lines refer to the estimated lower and upper bounds and the dashed lines refer to the real actual state. We see that the interval observer described by signals $\overline{x}, \underline{x}$ exhibits the upper and lower bounds, despite system uncertainties and input uncertain delay $d_a(t)$ all the time. In addition, it is an input to state stable interval estimator with the parameters $a = 2$ and $b = 1$ in (21) for the considered process.

As shown in Figure 6, one can see that despite the communication delay $d_m(t)$ in the measurement channel, two bounds of the plant state, described by the continuous lines, are provided by the predictor after a finite time $\overline{d} = 0.1s$. We can conclude that the predictor outputs share the same dynamics of the interval observer bounds, i.e. $\overline{z} \to \overline{x}$ and $\underline{z} \to \underline{x}$ (see Figure 5), despite uncertain delay $d_m(t)$. Consequently, the inclusion (29) is satisfied and the transmission delays $d_m(t)$ is compensated.

Finally, Figure 7 illustrates the state predictor stability and robustness. Lower predictor error $\underline{e} = \underline{z} - \underline{x}$ and upper predictor error $\overline{e} = \overline{z} - \overline{x}$ both tend to 0 when $\lim_{k\to\infty} t_k = \infty$, $\{k \in \mathbb{N}\}$. Thus, the theoretical developments are well validated by these simulation results.

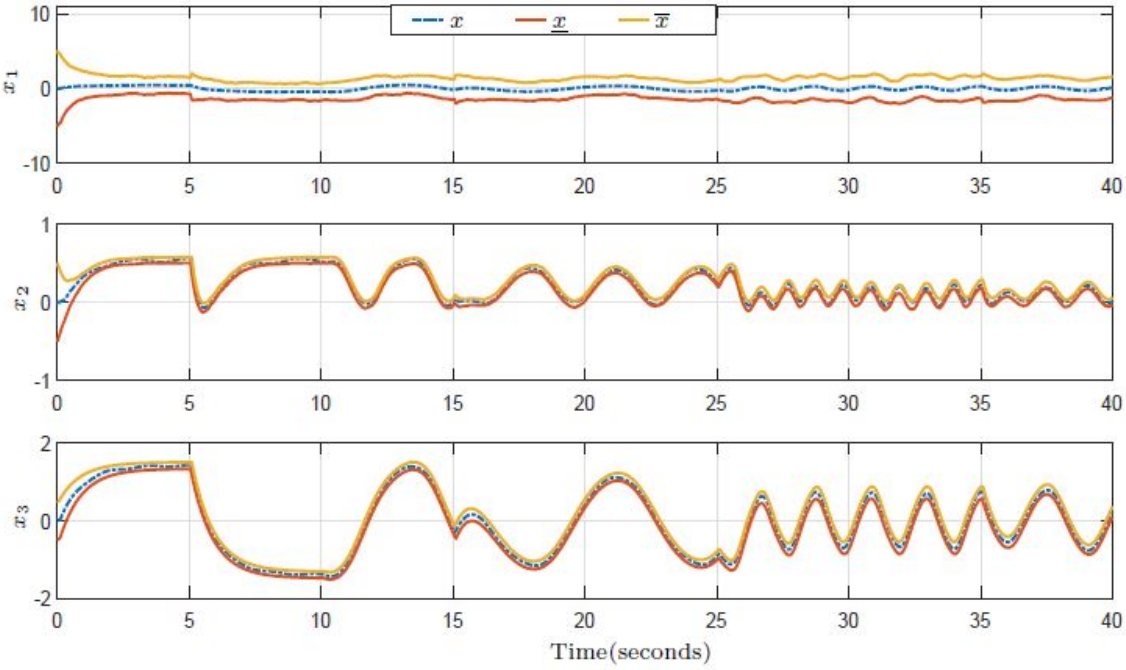


Figure 5. Evolution of the real state and the interval observer bounds.

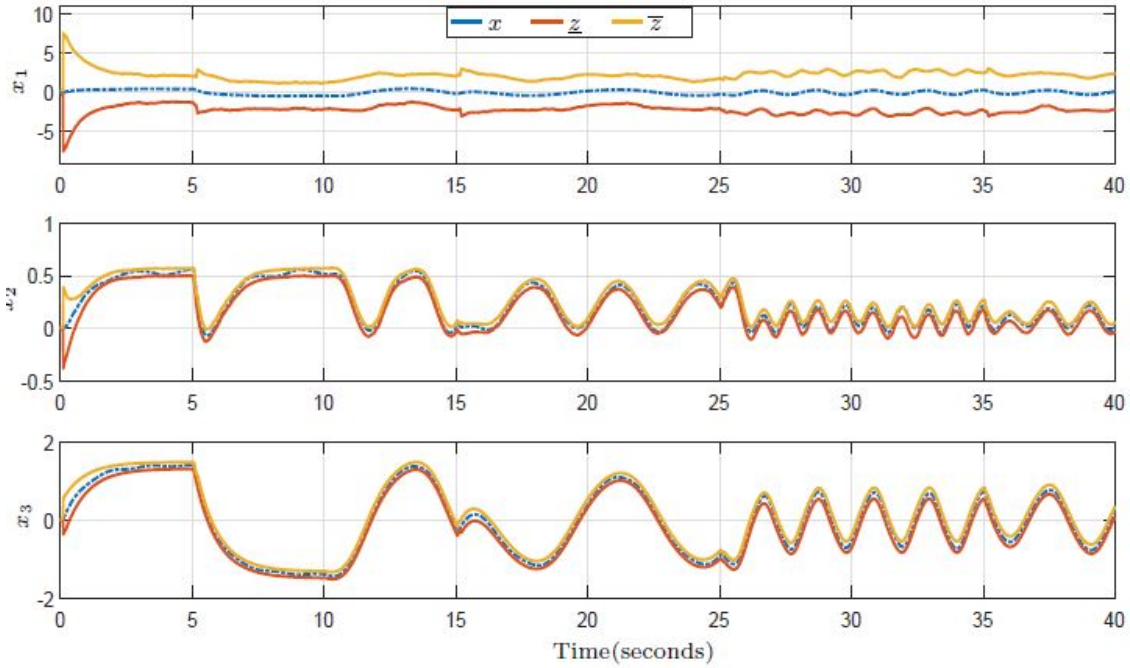


Figure 6. Evolution of the real state and the state predictor outputs.

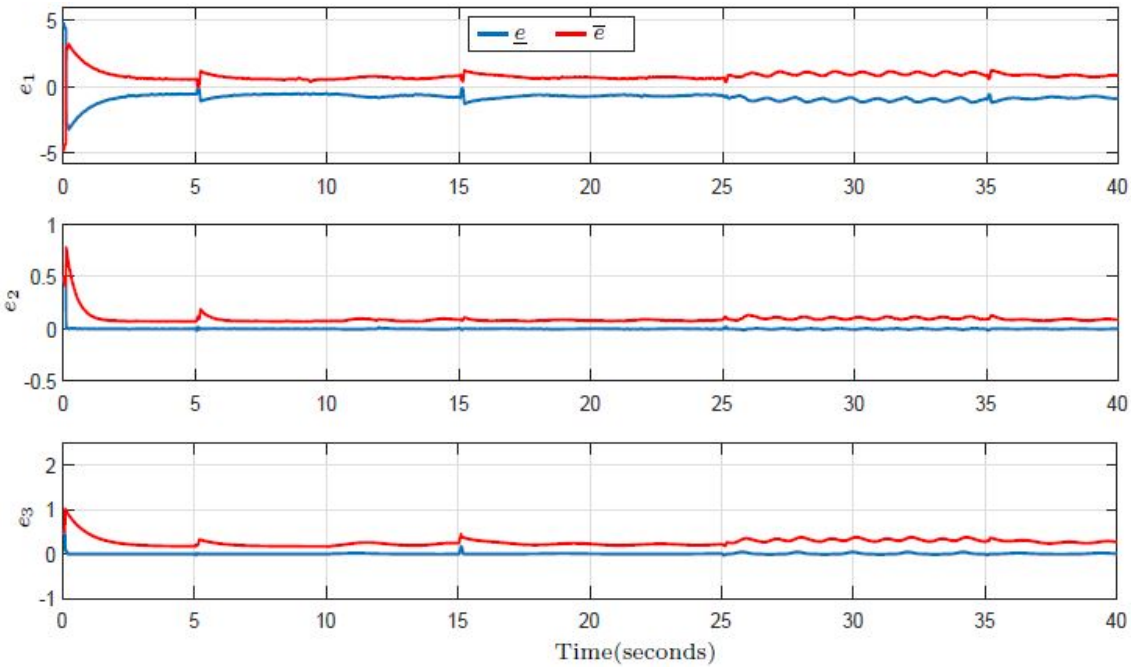


Figure 7. Evolution of the predictor errors.

7. Conclusion and open problems

Remote supervision problem of a class of NNCS with different kinds of uncertainties has been solved in this work. Under the assumption that external disturbances, measurement noises and input-delay are bounded, a local interval observer, using input-delay approach, has been developed. This set-membership estimation technique is conceptually different from standard pointwise observers, and the main advantage of an interval observer is that it guarantees estimation error values in the calculated bounds without requirements of point to point uncertainties dynamics. Moreover, interval observers are practical for estimating the transition of state variables of dynamical systems since they produces time-varying intervals in which the state variables are guaranteed to stay all times in the presence of large and fluctuating disturbances. Next, the designed predictor could successfully regenerate the estimated states despite uncertain network transportation delay. To the best of the author's knowledge, it is the first time that interval observer/predictor has been proposed for nonlinear networked control systems which are affected by many sources of uncertainties. Despite these important contributions, it is worth mentioning that some limitations are present. First, the use of a transformation of coordinates is rather restrictive in the set-membership framework. Second, the simulations results are given from an academic system. On the other hand, other network constraints such as packet lost, quantization problems are not studied in our structure. All those limitations will be considered in forthcoming works. Furthermore, the challenging problem of stabilization of such class of NNCS is also an interesting subject to be investigated.

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