POD-DEIM Model Order Reduction in the Case of Damaging RC Structures Subjected to Earthquakes

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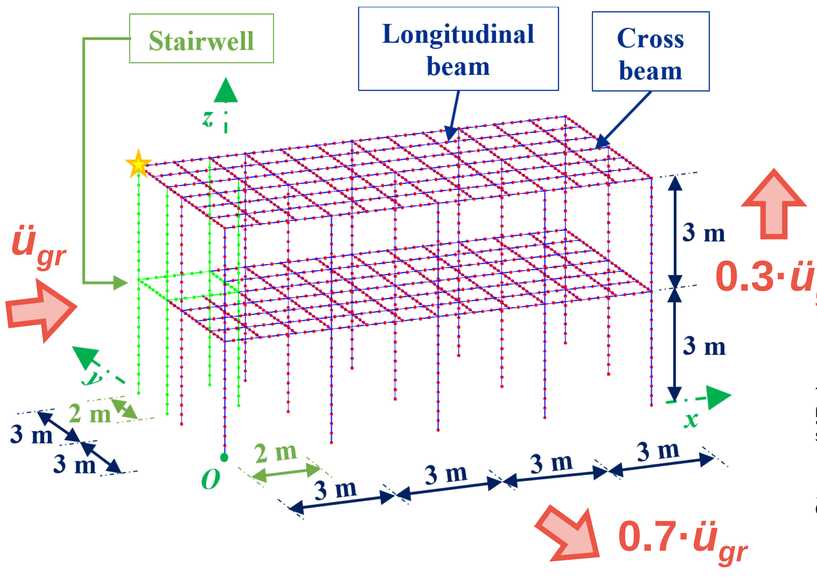
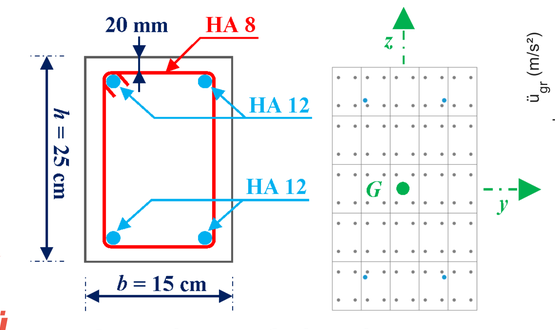
Earthquake Engineering, Reinforced Concrete, Multifibre Beam Elements, Proper Orthogonal Decomposition (POD), Discrete Empirical Interpolation Method (DEIM)

In this work, a POD-DEIM reduced order model is used with a non-iterative α-OS time-integration scheme to speed up FEM analyses on a two-storey RC structure made of damaging multifibre beam elements subjected to a seismic hazard modelled by a set of 12 earthquakes.

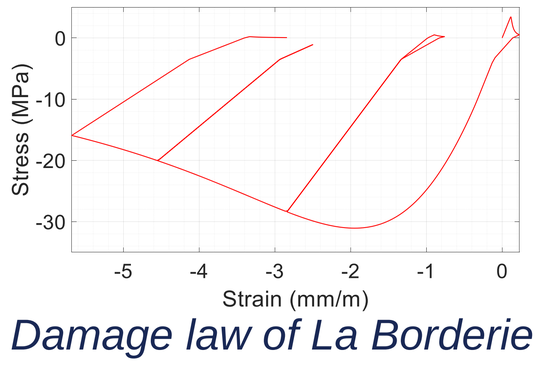
Case study



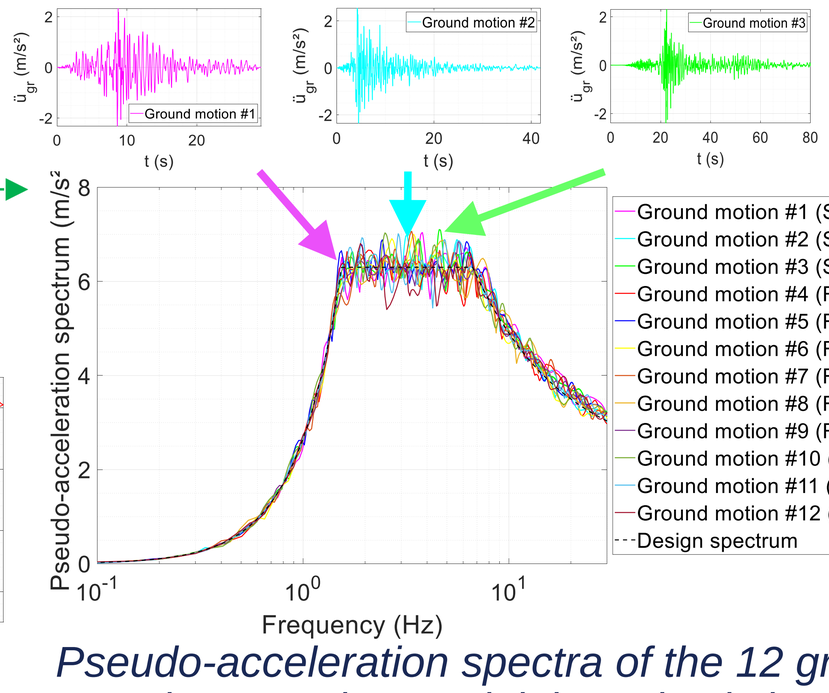
Two storey multifibre RC building made of 9222 free DOFs



Section of the beams



Damage law of La Borderie



Pseudo-acceleration spectra of the 12 ground motions used to model the seismic hazard

*Full order FEM analyses with ground motions **#1 to #3***

Snapshots

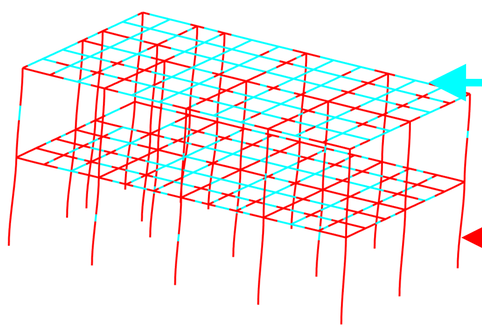
Non iterative α-OS time scheme

$$\boldsymbol{M} \cdot \ddot{\boldsymbol{u}} + \boldsymbol{C} \cdot \dot{\boldsymbol{u}} + \boldsymbol{K}_E \cdot \boldsymbol{u} + \tilde{\boldsymbol{r}}_{NL}(\tilde{\boldsymbol{u}}) = -\boldsymbol{M} \cdot \Gamma \cdot \ddot{u}_{gr}(t)$$

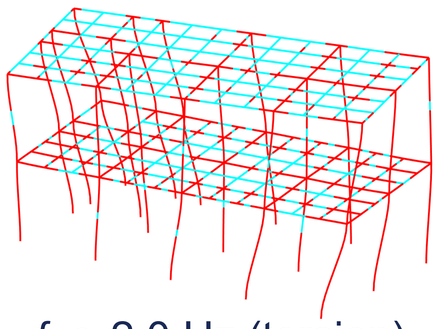
Explicit prediction of the nonlinear part of the restoring force vector

*SVD on a set of snapshots of **u***

POD-modes



$f_1 \approx 1.4$ Hz (bending)



$f_3 \approx 2.0$ Hz (torsion)

$$\boldsymbol{u} \approx \boldsymbol{B} \cdot \boldsymbol{q}$$

*Projection of the displacements on a basis **B** made of n POD-modes*

SVD on a set of snapshots of $\tilde{\boldsymbol{r}}_{NL}$

DEIM-modes

Domain where the restoring forces are interpolated

Reduced integration domain (RID)

Basis made of m DEIM-modes

$$\boldsymbol{A} = \boldsymbol{\Psi} \cdot (\boldsymbol{P}^T \cdot \boldsymbol{\Psi})^{-1}$$

Partition matrix related to m components of the RID

Interpolation operator

$$\tilde{\boldsymbol{r}}_{NL} \approx \boldsymbol{A} \cdot \tilde{\boldsymbol{r}}_{NL}^{RID}$$

Interpolation of the restoring force vector on the full domain

Online phases

$$1 - \frac{\sum_{i=1}^{n} \lambda_i}{\sum_{i=1}^{N} \lambda_i} \leq \varepsilon$$

POD : $\varepsilon = 1 \cdot 10^{-2}$ → ***n* = 20**
DEIM : $\varepsilon = 5 \cdot 10^{-6}$ → ***m* = 2303**

Criterion used to select the number of modes

Ground-motion #11

Hor. disp. of the N-W corner

Time savings with the POD and the POD-DEIM

Ground motion	#4	#5	#6	#7	#8	#9	#10	#11	#12
Error (%)	0.3	0.6	0.2	0.6	0.3	0.7	0.3	0.2	0.6
Time saving POD (%)	22	28	23	27	27	27	23	30	27
Time saving POD-DEIM (%)	46	49	47	48	48	48	48	49	48

The error is less than 1 % with a 50 % time saving during each online phase.

*Reduced order FEM analyses with ground motions **#4 to #12***

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References

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