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Abstract: In this work, we consider a discrete-time switched linear system. A novel approach is introduced to estimate its state in a finite fixed time which can be arbitrarily chosen and is independent of the initial state. For cases where the additive disturbance and measurement noise are known, we provide an exact estimation. Otherwise, a finite-time interval observer is designed. The crucial idea is based on using past values of the input and output of the studied system and a minimal dwell time condition. Simulation results are provided to illustrate the effectiveness of the proposed techniques in different scenarios of finite time choices.

Keywords: Exact estimation, approximate estimation, finite-time observers, discrete-time, switched linear systems.

1. INTRODUCTION

Switched systems are an important class of hybrid dynamical systems. They appear in several fields and can model a lot of complex real-life phenomena. They consist of a family of discrete- or continuous-time subsystems and a switching rule that operates switching between them. It has witnessed an increasing interest in the literature where stability and control problems for switched systems have been widely investigated during the past years, see for instance (Serres et al. (2010), Liberzon and Morse (1999)).

State estimation of switched systems has been studied in a good deal of works but still forms an interesting research topic (see e.g., Yang et al. (2017), Lin and Gao (2015), Ethabet et al. (2018), Ríos et al. (2012), Dinh et al. (2020) and the reference therein). These works address the problem of state estimation referred to asymptotic converging observers where the state observation error is asymptotically stable but generally there is no information on the convergence time. However, in practice, many applications require some transient performances, e.g., guaranteeing the finite-time convergence of the estimation error. For this requirement, cutting-edge finite-time estimation methods have been intensively designed. The authors in Dinh et al. (2019) proposed a finite-time observer for discrete-time systems subject to disturbances where the system state can be determined using two sub-dynamics and a single delay. A fault reconstruction method was proposed in Lee and Park (2012) using a finite-time converging unknown input observer. In Raff et al. (2006), a finite-time observer was designed for linear systems with unknown inputs. A finite-time convergence analysis based on a Lyapunov function and a finite-time observer for singular systems under unknown inputs were introduced in Polyakov and Poznyak (2009) and Zhang et al. (2019), respectively. Most of aforementioned results on finite-time observer focus on continuous-time systems. In the context of finite/fixed-time observer design for switched systems, some recent works can be cited here, for example in Chen et al. (2019), finite-time state estimation and active mode identification were proposed for uncertain continuous-time switched linear systems while a fixed-time observer was constructed for a class of continuous-time switched nonlinear systems in Dinh and Defoort (2020). To the best of the authors knowledge, the case of discrete-time switched systems has not been fully considered in the literature, which motivates our research.

In this paper, motivated by the preliminary work Dinh and Defoort (2020) based on observers with delays, we extend the technique for a class of uncertain discrete-time switched linear systems. Two objectives can be achieved. First, without uncertainties, a finite-time observer is designed, which can exactly estimate the system state for a preset convergence time. Second, under unknown but bounded uncertainties, a lower and an upper bound of the state are provided after this finite time.

The paper is structured as follows. Some preliminaries are introduced in Section 2. In Section 3, the problem statement is provided. Finite-time exact estimation for discrete-time switched linear systems with known disturbances is developed in Section 4. In Section 5, a lower and an upper bound for the solutions in the presence of bounded unknown disturbances are constructed. In Section 6, simulation results are shown to illustrate the efficiency of

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the proposed technique. Finally, the paper is concluded in Section 7.

2. PRELIMINARIES

In the sequel, the following notations are adopted. $\mathbb{R}$ and $\mathbb{N}$ denote respectively the sets of real and natural numbers. $|x|$ denotes the Euclidean norm of a vector $x \in R^n$. $\overline{1, N}$ is a sequence of integers $1, ..., N$. Let $\underline{x}$ and $\overline{x}$ be respectively the lower and upper bounds of a variable x such that $\underline{x} \leq x \leq \overline{x}$. For a matrix $A \in \mathbb{R}^{m \times n}$, let $A^+ = \max\{0, A\}$ and $A^- = A^+ - A$.
Throughout this paper, the comparison operators $\leq$ and $\geq$ should be understood elementwise for vectors and matrices.

Lemma 1. Chebotarev et al. (2013) consider a vector $x \in \mathbb{R}^n$ satisfying $\underline{x} \leq x \leq \overline{x}$ and a constant matrix $A \in \mathbb{R}^{m \times n}$, then

$$A^+\underline{x} - A^-\overline{x} \leq Ax \leq A^+\overline{x} - A^-\underline{x}. \tag{1}$$

3. PROBLEM FORMULATION

Consider the following discrete-time switched linear system

$$\begin{cases} x(k+1) = A_q x(k) + B_q u(k) + w(k) \\ y(k) = C_q x(k) + v(k) \end{cases}, q \in \overline{1, N},\ N \in \mathbb{N} \tag{2}$$

where $x \in \mathbb{R}^{n_x}$ is the state, $y \in \mathbb{R}^{n_y}$ is the output, $u \in \mathbb{R}^{n_u}$ is the input, $w \in \mathbb{R}^{n_w}$ is the state disturbance and $v \in \mathbb{R}^{n_v}$ is the measurement noise. A_q, B_q and C_q are constant matrices of appropriate dimensions. The index q specifies, at each instant, the system that is currently followed and N is the number of mode. Let $\{t_q\}$ denote the switching time instants. In this paper, it is supposed that the active subsystem and the switching instants are known. The following assumptions will be considered in this paper.

Assumption 1. The system (2) satisfies the minimal dwell time condition and the dwell time is a known constant. That means there exists a known positive constant τ_a such that the switching times $\{t_q\}$ satisfy for all $q \in \overline{1, N}$

$$t_q - t_{q-1} \geq \tau_a.$$

Assumption 2. $\forall q \in \overline{1, N}$, the pairs (A_q, C_q) are observable and A_q are invertible.

Assumption 2 means that there exist a strictly positive integer $h < \tau_a$, and matrices $L_q \in \mathbb{R}^{n_x \times n_y}$ such that for all $q \in \overline{1, N}$, the matrices $H_q^{-h} - A_q^{-h}$ are invertible where

$$H_q = A_q + L_q C_q \in \mathbb{R}^{n_x \times n_x}. \tag{3}$$

Assumption 3. The measurement noise and the state disturbance are supposed to be unknown but bounded with a priori known bounds such that

$$\underline{w} \leq w(k) \leq \overline{w} \tag{4}$$

$$\underline{v} \leq v(k) \leq \overline{v} \tag{5}$$

where $\overline{w} \in \mathbb{R}^{n_w}$ and $\overline{v} \in \mathbb{R}^{n_v}$

In the following, the aim is to study the finite-time observer design for (2) in two scenarios: (i) first, we propose an exact finite-time observer design for (2) when the disturbances and measurement noise are known; (ii) secondly, we deal with a more general case where the uncertainties are supposed to be unknown but bounded, by generating a lower and an upper bound for the system state of (2) after a finite time. It should be noted that the key ideas are inspired by Dinh and Defoort (2020) with an extension to discrete-time switched cases.

4. EXACT STATE ESTIMATION FOR DISCRETE-TIME SWITCHED LINEAR SYSTEMS

In this section, exact estimation approach is proposed for (2) subject to known disturbances and measurement noise. This section provides the exact estimation of the system dynamics; i.e., the state estimation error equals to zero in finite time. Let us state the following theorem which exactly compute the state after a finite time which is an artificial delay.

Theorem 1. For the system (2), let Assumptions 1 and 2 be satisfied. Let $L_q \in \mathbb{R}^{n_x \times n_y}$ and $h \in \mathbb{N}$, $1 \leq h < \tau_a$ be such that the matrices $H_q^{-h} - A_q^{-h}$, $\forall q \in \overline{1, N}$ are invertible. Then, for a given input u, the state of (2) satisfies, for all $q \in \overline{1, N}$ in each $[t_{q-1} + h, t_q]$,

$$\begin{aligned} x(k) = &-E_{q_h} \sum_{l=k-h}^{k-1} A^{k-h-l-1} B_q u(k) \\ &+E_{q_h} \sum_{l=k-h}^{k-1} H^{k-h-l-1} [B_q u(k) - L_q y(k)] \\ &-E_{q_h} \sum_{l=k-h}^{k-1} A^{k-h-l-1} w(k) \\ &+E_{q_h} \sum_{l=k-h}^{k-1} H^{k-h-l-1} [w(k) + L_q v(k)], \end{aligned} \tag{6}$$

where

$$E_{q_h} = (H_q^{-h} - A_q^{-h})^{-1}. \tag{7}$$

Proof. From the output y and the definition of H_q, for all $k \in [t_{q-1}+h, t_q]$, $q \in \overline{1, N}$, we observe that the system (2) can be rewritten in two different representations as follows

$$x(k+1) = A_q x(k) + B_q u(k) + w(k) \tag{8}$$

$$\begin{aligned} x(k+1) = H_q x(k) + B_q u(k) - L_q y(k) \\ +w(k) + L_q v(k) \end{aligned} \tag{9}$$

Combining equations (8) and (9) between two values $c_1, c_2 \in \mathbb{N}$ such that $c_2 \leq c_1$, we obtain the equalities:

$$\begin{aligned} x(c_1) = A_q^{c_1 - c_2} x(c_2) + \sum_{l=c_2}^{c_1 - 1} A_q^{c_1 - l - 1} [B_q u(k) + \\ +w(k)] \end{aligned} \tag{10}$$

$$\begin{aligned} x(c_1) = H_q^{c_1 - c_2} x(c_2) + \sum_{l=c_2}^{c_1 - 1} H_q^{c_1 - l - 1} [B_q u(k) - L_q y(k) \\ +w(k) + L_q v(k)] \end{aligned} \tag{11}$$

Now, for all $k \in [t_{q-1} + h, t_q]$, $q \in \overline{1, N}$, selecting $c_1 = k$ and $c_2 = k - h$. The above equalities give

$$x(k) = A_q^h x(k-h) + \sum_{l=k-h}^{k-1} A_q^{k-l-1}\left[B_q u(k) + w(k)\right] \quad (12)$$

$$x(k) = H_q^h x(k-h) + \sum_{l=k-h}^{k-1} H_q^{k-l-1}[B_q u(k) - L_q y(k) + w(k) + L_q v(k)] \quad (13)$$

then for all $k \in [t_{q-1}+h, t_q]$, $q \in \overline{1,N}$, we obtain

$$(H_q^{-h} - A_q^{-h})x(k) = -\sum_{l=k-h}^{k-1} A_q^{k-h-l-1}\left[B_q u(k) + w(k)\right] + \sum_{l=k-h}^{k-1} H_q^{k-h-l-1}\left[B_q u(k) - L_q y(k) + w(k) + L_q v(k)\right]$$

Since $H_q^{-h} - A_q^{-h}$, $\forall\, q \in \overline{1,N}$ are assumed to be invertible, we can deduce that (6) is satisfied for all $k \in [t_{q-1}+h, t_q]$, $q \in \overline{1,N}$.

Remark 1. Expression (6) can contain many terms because h can be large, so many values have to be stored. To avoid this problem, we propose a second alternative which is based on the use of dynamic extensions.

Theorem 2. Suppose that Assumptions 1 and 2 are satisfied and there exist $L_q \in \mathbb{R}^{n_x \times n_y}$ and $h \in \mathbb{N}$, $1 \leq h < \tau_a$ such that the matrices $H_q^{-h} - A_q^{-h}$, $\forall\, q \in \overline{1,N}$ are invertible. Consider the N dynamic extensions, each one associated to a different subsystem defined for all $q \in \overline{1,N}$ as follows

$$\hat{x}(k+1) = A_q \hat{x}(k) + B_q u(k) + w(k) \quad (14)$$

and

$$x_*(k+1) = H_q x_*(k) + B_q u(k) - L_q y(k) + w(k) + L_q v(k). \quad (15)$$

Then, for a given input u, the state of (2) in each $[t_{q-1}+h, t_q]$ satisfies, for all $q \in \overline{1,N}$,

$$x(k) = E_{q_h}\left[H_q^{-h} x_*(k) - x_*(k-h) - A_q^{-h}\hat{x}(k) + \hat{x}(k-h)\right] \quad (16)$$

Proof. Consider $(\hat{x}, x_*)$ a solution of the system ((14), (15)) and the solution x of (2). Then, similar to Proof 4, we have

$$\hat{x}(c_1) = A_q^{c_1-c_2}\hat{x}(c_2) + \sum_{l=c_2}^{c_1-1} A_q^{c_1-l-1}\left[B_q u(k) + w(k)\right] \quad (17)$$

$$x_*(c_1) = H_q^{c_1-c_2} x_*(c_2) + \sum_{l=c_2}^{c_1-1} H_q^{c_1-l-1}[B_q u(k) - L_q y(k) + w(k) + L_q v(k)] \quad (18)$$

for all $k \in [t_{q-1}+h, t_q]$, $q \in \overline{1,N}$. It follows that

$$\sum_{l=k-h}^{k-1} A^{k-h-l-1}\left[B_q u(k) + w(k)\right] = A_q^{-h}\hat{x}(k) - \hat{x}(k-h) \quad (19)$$

$$\sum_{l=k-h}^{k-1} H_q^{k-h-l-1}\left[B_q u(k) - L_q y(k) + w(k) + L_q v(k)\right] = H_q^{-h} x_*(k) - x_*(k-h) \quad (20)$$

Finally, from (6), (19) and (20), we can deduce that for all $k \in [t_{q-1}+h, t_q]$, $q \in \overline{1,N}$,

$$E_{q_h}^{-1} x(k) = H_q^{-h} x_*(k) - x_*(k-h) - A_q^{-h}\hat{x}(k) - \hat{x}(k-h) \quad (21)$$

Then, we deduce that (16) holds.

Remark 2. The approach described above requires that the disturbances and the measurement noise are known. However, in most of cases, it is not trivial to obtain this information. In the next, a technique is carried out to overcome this limitation by assuming that the disturbances are unknown and only bounded by known bounds, which is acceptable in many applications.

5. APPROXIMATIVE STATE ESTIMATION FOR DISCRETE-TIME SWITCHED SYSTEMS

This section proposes a novel approach to estimate the system state of (2) subject to unknown disturbances and measurement noise. We suppose that Assumptions 1-3 are satisfied. Let us introduce the following matrices and vectors used in sequel.

$$F_{q_h} = -E_{q_h} A_q^{-(h+1)}, \qquad G_{q_h} = E_{q_h} H_q^{-(h+1)} \quad (22)$$

$$w_L = \left(\sum_{l=1}^{h} F_{q_h} A_q^l + G_{q_h} A_q^l\right)^+ \overline{w} - \left(\sum_{l=1}^{h} F_{q_h} A_q^l + G_{q_h} A_q^l\right)^- \underline{w}, \quad (23)$$

$$w_S = \left(\sum_{l=1}^{h} F_{q_h} A_q^l + G_{q_h} A_q^l\right)^+ \underline{w} - \left(\sum_{l=1}^{h} F_{q_h} A_q^l + G_{q_h} A_q^l\right)^- \overline{w}, \quad (24)$$

$$v_L = \left(G_{q_h}\sum_{l=1}^{h} H_q^l L_q\right)^+ \overline{v} - \left(G_{q_h}\sum_{l=1}^{h} H_q^l L_q\right)^- \underline{v}, \quad (25)$$

$$v_S = \left(G_{q_h}\sum_{l=1}^{h} H_q^l L_q\right)^+ \underline{v} - \left(G_{q_h}\sum_{l=1}^{h} H_q^l L_q\right)^- \overline{v}, \quad (26)$$

The following theorem ensures in finite time an approximate state estimation. A lower and an upper bound are calculated to enclose the real state. Let us state the following result.

Theorem 3. For the system (2), assume that Assumption 1-3 are satisfied. Let $L_q \in \mathbb{R}^{n_x \times n_y}$ and $h \in \mathbb{N}$, $1 \leq h < \tau_a$ be such that $\forall\, q \in \overline{1,N}$, $H_q^{-h} - A_q^{-h}$ are invertible. For a given input u, the solution x of the system (2) satisfies for all $k \in [t_{q-1}+h, t_q]$ and $\forall q \in \overline{1,N}$,

$$\underline{x}(k) \leq x(k) \leq \overline{x}(k) \tag{27}$$

where

$$\begin{aligned}\overline{x}(k) = &-E_{q_h}\sum_{l=k-h}^{k-1} A_q^{k-h-l-1} B_q u(k)\\ &+E_{q_h}\sum_{l=k-h}^{k-1} H_q^{k-h-l-1}\left[B_q u(k) - L_q y(k)\right]\\ &+d_L + v_L,\end{aligned} \tag{28}$$

$$\begin{aligned}\underline{x}(k) = &-E_{q_h}\sum_{l=k-h}^{k-1} A_q^{k-h-l-1} B_q u(k)\\ &+E_{q_h}\sum_{l=k-h}^{k-1} H_q^{k-h-l-1}\left[B_q u(k) - L_q y(k)\right]\\ &+d_S + v_S.\end{aligned} \tag{29}$$

Proof. From (6), we have

$$\begin{aligned}x(k) = &-E_{q_h}\sum_{l=k-h}^{k-1} A_q^{k-h-l-1} B_q u(k)\\ &+E_{q_h}\sum_{l=k-h}^{k-1} H_q^{k-h-l-1}[B_q u(k) - L_q y(k)]\\ &-E_{q_h}A_q^{-(h+1)}\sum_{l=k-h}^{k-1} A_q^{k-l} w(k)\\ &+E_{q_h}H_q^{-(h+1)}\sum_{l=k-h}^{k-1} H_q^{k-l}[w(k) + L_q v(k)]\end{aligned} \tag{30}$$

Using the expressions of F_{q_h} and G_{q_h}, it follows that for all $k \in [t_{q-1}+h, t_q]$, $q \in \overline{1,N}$,

$$\begin{aligned}x(k) = &-E_{q_h}\sum_{l=k-h}^{k-1} A_q^{k-h-l-1} B_q u(k)\\ &+E_{q_h}\sum_{l=k-h}^{k-1} H_q^{k-h-l-1}[B_q u(k) - L_q y(k)]\\ &+\sum_{l=1}^{h}(F_{q_h}A_q^l + G_{q_h}H_q^l) w(k-l)\\ &+G_{q_h}\sum_{l=1}^{h} H_q^l v(k-l)\end{aligned} \tag{31}$$

Bearing in mind Assumption 3 and Lemma 1, we obtain, for all $k \in [t_{q-1}+h, t_q]$ and $q \in \overline{1,N}$,

$$\begin{aligned}&(\sum_{l=1}^{h} F_{q_h}A_q^l + G_{q_h}A_q^l)^+ \underline{w} - (\sum_{l=1}^{h} F_{q_h}A_q^l + G_{q_h}A_q^l)^- \overline{w}\\ &\leq \sum_{l=1}^{h}(F_{q_h}A_q^l + G_{q_h}A_q^l) w(k-l)\\ &\leq (\sum_{l=1}^{h} F_{q_h}A_q^l + G_{q_h}A_q^l)^+ \overline{w} - (\sum_{l=1}^{h} F_{q_h}A_q^l + G_{q_h}A_q^l)^- \underline{w}.\end{aligned}$$

$$\begin{aligned}&(G_{q_h}\sum_{l=1}^{h} H_q^l L_q)^+ \underline{v} - (G_{q_h}\sum_{l=1}^{h} H_q^l L_q)^- \overline{v}\\ &\leq G_{q_h}\sum_{l=1}^{h} H_q^l L_q v(k-l)\\ &\leq (G_{q_h}\sum_{l=1}^{h} H_q^l L_q)^+ \overline{v} \ - \ (G_{q_h}\sum_{l=1}^{h} H_q^l L_q)^- \underline{v}.\end{aligned}$$

Thus,

$$d_S \leq \sum_{l=1}^{h}(F_{q_h}A_q^l + G_{q_h}A_q^l) w(k-l) \leq d_L \tag{32}$$

$$v_S \leq G_{q_h}\sum_{l=1}^{h} H_q^l L_q v(k-l) \leq v_L \tag{33}$$

where d_S, d_L, v_S and v_L are given from (23) to (26). From (31), (32) and (33), we deduce that (27) hold.

Based on the motivation of proposing Theorem 2 to avoid to have to store many values, the following theorem is introduced. It provides an approximate state estimation of (2) in term of dynamic extensions.

Theorem 4. For the system (2), let the conditions introduced in Theorem 3 be satisfied. For a given input u and let x be a solution of (2) defined over $\mathbb{N}$. Consider the N dynamic extensions, each one associated to a different subsystem

$$z_a(k+1) = A_q z_a(k) + B_q u(k) \tag{34}$$

$$z_h(k+1) = H_q z_h(k) + B_q u(k) - L_q y(k). \tag{35}$$

Then, for all $k \in [t_{q-1}+h, t_q]$, $q \in \overline{1,N}$, the following inequalities

$$\underline{\mathrm{X}}(Z_k) \leq x(k) \leq \overline{\mathrm{X}}(Z_k) \tag{36}$$

hold, where $Z = (z_a, z_h)$ and the two estimated bounds of the system state i.e., $\underline{\mathrm{X}}$ and $\overline{\mathrm{X}}$, are given by

$$\begin{aligned}\overline{\mathrm{X}}(Z_k) = E_{q_h}[&z_a(k-h) - A_q^{-h} z_a(k) + H_q^{-h} z_h(k)\\ &-z_h(k-h)] + d_L + v_L\end{aligned} \tag{37}$$

$$\begin{aligned}\underline{\mathrm{X}}(Z_k) = E_{q_h}[&z_a(k-h) - A_q^{-h} z_a(k) + H_q^{-h} z_h(k)\\ &-z_h(k-h)] + d_S + v_S\end{aligned} \tag{38}$$

with d_S, d_L, v_S and v_L are respectively defined in (23), (24), (25) and (26).

Proof. Consider a solution x of the system (2). For all $k \in [t_{q-1}+h, t_q]$ and $q \in \overline{1,N}$, we have

$$z_a(k) = A_q^h z_a(k-h) + \sum_{l=k-h}^{k-l} A_q^{k-l-1} B_q u(k), \tag{39}$$

$$x_h(k) = H_q^h z_h(k-h) + \sum_{l=k-h}^{l-l} H_q^{k-l-1}[B_q u(k) - L_q y(k)]. \tag{40}$$

Equations (39) and (40) can be rewritten as follows

$$\sum_{l=k-h}^{k-1} A^{k-h-l-1} B_q u(k) = A_q^{-h} z_a(k) - z_a(k-h) \tag{41}$$

$$\sum_{l=k-h}^{k-1} H_q^{k-h-l-1}\left[B_q u(k) - L_q y(k)\right] \\ = H_q^{-h} z_h(k) - z_h(k-h) \tag{42}$$

Theorem 3 ensures that the inequalities in (27) are satisfied. Hence, using (41) and (42) implies that (36) is satisfied.

Remark 3. Through suitable dynamics, Theorem 2 and Theorem 4 give the exact and approximate estimation of the state of (2) in finite time which is independent of initial conditions of (2). Between two switching instants and after a known time h, while Theorem 2 proposes an estimator such that the estimation error exactly equals to zero, the Theorem 4 computes an interval for the state. It is worth noticing that h can be chosen in advance but must be less than the minimal dwell time τ_a introduced in Assumption 1.

6. SIMULATION RESULTS

The numerical example is defined with two subsystems i.e., $N = 2$ as follows

$$\begin{cases} x(k+1) = A_q x(k) + B_q u(k) + w(k) \\ y(k) = C_q x(k) + v(k) \end{cases} \quad q \in \overline{1,2} \tag{43}$$

where

$$A1 = \begin{bmatrix} 0.3 & -2 \\ 0 & 0.6 \end{bmatrix}, \quad A2 = \begin{bmatrix} 0.5 & -1.1 \\ 0 & 0.16 \end{bmatrix}$$

$$B1 = \begin{bmatrix} 6 \\ 1 \end{bmatrix}, \quad B2 = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

$$C1 = [1 \quad 0], \quad C2 = [0.1 \quad 1]$$

$$w(k) = 0.05 \begin{bmatrix} \sin(0.1k) \\ \sin(0.1k) \end{bmatrix} \quad and \quad v(k) = 0.2\sin(0.1k).$$

Under Assumption 1, let us choose $L_1 = [0.01 \ \ -0.01]^T$ and $L_2 = [0.02 \ \ -0.01]^T$, then

$$H_1 = A_1 + L_1 C_1 = \begin{pmatrix} 0.31 & -2 \\ -0.01 & 0.6 \end{pmatrix}$$

and

$$H_2 = A_2 + L_2 C_2 = \begin{pmatrix} 0.502 & -1.08 \\ -0.001 & 0.15 \end{pmatrix}.$$

Now, we define

$$R_{11} = \begin{pmatrix} 0.15 & 1 \\ 0 & 1 \end{pmatrix}, \quad S_{11} = \begin{pmatrix} 0.3 & 0 \\ 0 & 0.6 \end{pmatrix},$$

$$R_{21} = \begin{pmatrix} -0.301 & 1 \\ 0 & 1 \end{pmatrix}, \quad S_{21} = \begin{pmatrix} 0.5 & 0 \\ 0 & 0.16 \end{pmatrix}.$$

Then for $q \in \overline{1,2}$, $R_{q1} A_q R_{q1}^{-1} = S_{q1}$. Similarly, $R_{q2} H_q R_{q2}^{-1} = S_{q2}$, for $q \in \overline{1,2}$ with

$$R_{12} = \begin{pmatrix} 1 & 5.75 \\ 1 & -34.75 \end{pmatrix}, \quad S_{12} = \begin{pmatrix} 0.2525 & 0 \\ 0 & 0.6575 \end{pmatrix},$$

$$R_{22} = \begin{pmatrix} 1 & -3 \\ 1 & 355 \end{pmatrix}, \quad S_{22} = \begin{pmatrix} 0.505 & 0 \\ 0 & 0.147 \end{pmatrix}.$$

For an integer $h > 0$ the matrices $H_q^{-h} - A_q^{-h}$ can be calculated as follows

$$H_q^{-h} - A_q^{-h} = R_{q2}^{-1} S_{q2}^{-h} R_{q2} - R_{q1}^{-1} S_{q1}^{-h} R_{q1} \\ = \begin{pmatrix} \alpha_{q1} & \alpha_{q2} \\ \alpha_{q3} & \alpha_{q4} \end{pmatrix}$$

with

$$\alpha_{11} = 0.858(0.2525)^{-h} + 0.142(0.6575)^{-h} - (0.3)^{-h},$$
$$\alpha_{12} = 4.9336(0.2525)^{-h} - 4.9336(0.6575)^{-h} \\ -6.6666(0.3)^{-h} + 6.6666(0.6)^{-h},$$
$$\alpha_{13} = 0.0247(0.2525)^{-h} - 0.0247(0.6575)^{-h},$$
$$\alpha_{14} = 0.1419(0.2525)^{-h} + 0.858(0.6575)^{-h} - (0.6)^{-h},$$

and

$$\alpha_{21} = 0.9916(0.505)^{-h} + 0.0084(0.147)^{-h} - (0.5)^{-h},$$
$$\alpha_{22} = -2.9748(0.505)^{-h} + 2.9748(0.147)^{-h} \\ +3.2352(0.5)^{-h} - 3.2352(0.16)^{-h},$$
$$\alpha_{23} = -0.0028(0.505)^{-h} + 0.0028(0.147)^{-h},$$
$$\alpha_{24} = 0.0084(0.505)^{-h} + 0.9916(0.147)^{-h} - (0.16)^{-h}.$$

We can check that the matrices $H_1^{-h} - A_1^{-h}$ and $H_2^{-h} - A_2^{-h}$ are invertible for all $h \geq 2$. The Assumption 2 is satisfied.

In order to provide fast exact estimation, we set the convergence time $h = 2$ and $h = 3$. The finite time observer can be designed in the form given in Theorem 2. For the simulation, we set the initial condition $x(0) = [\,1 \ 1\,]^T$ and the input signal $u = 1$. The simulation results are showed in Fig. 1, Fig. 2.

Next, two bounds given in (37)-(38) are implemented. Fig. 3 illustrates the approximate state estimation for $h = 4$. For simulation, we set the initial condition $x(0) = [\,2 \ 1\,]^T$ and the input $u = 1$. We choose the following known bounds for disturbances w and measurement noise v: $\overline{w} = [0.05 \ 0.05]^T$, $\underline{w} = [0.05 \ 0.05]^T$, $\overline{v} = 0.2$ and $\underline{v} = -0.2$. The switching signal is plotted in Fig. 4, the minimum dwell time constraint is satisfied.

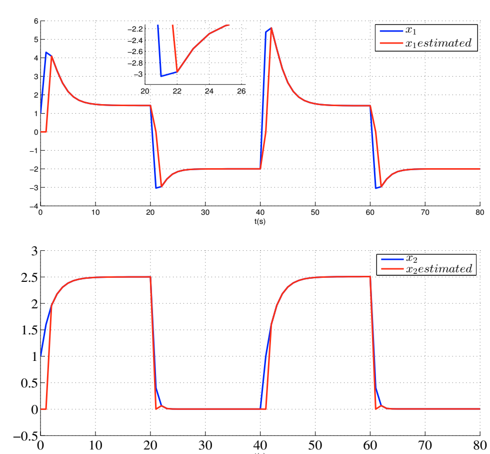

Fig. 1. Real value and exact estimation for $h = 2$.

The results in Fig. 1-2 provide the exact value of the state after respective finite times $h = 2$ and $h = 3$ for all k such that $t_{q-1} + h \leq k \leq t_q$, $q \in \overline{1,2}$. The results in Fig. 3 demonstrates the effectiveness of the second approach: the

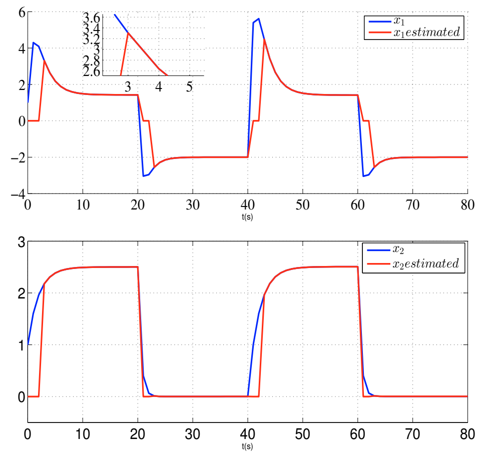

Fig. 2. Real value and exact estimation for $h = 3$.

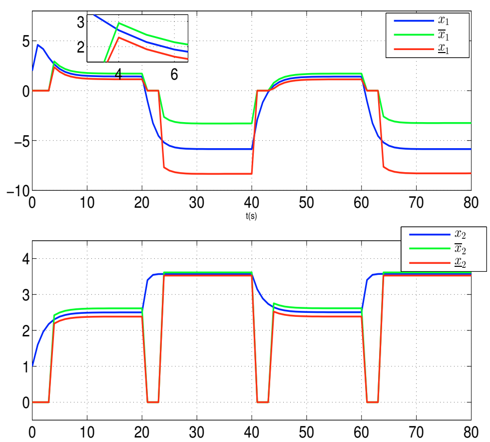

Fig. 3. Lower and upper bounds of the real state for $h = 4$

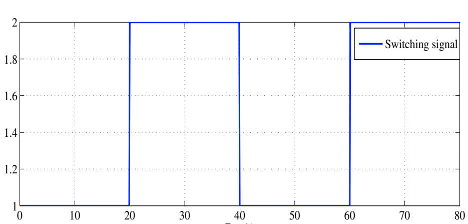

Fig. 4. Switching signal.

real state is framed by the upper and the lower trajectory after a finite time $h = 4$ in each interval $[t_{q-1} + h, t_q]$, $q \in \overline{1, 2}$.

7. CONCLUSION

This work focused on finite-time state estimation for discrete-time switched linear systems subject to disturbances and measurement noise. Two approaches were proposed to compute either the exact value or two bounds for the real state in finite time depending on the information of uncertain terms that we can obtain or not. Extensions to more general classes of discrete-time switched systems can be considered for future works. Another promising perspective is design of finite-time estimation with optimal gain choices.

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